UNIT 2 :
Algebra

4 Linear Inequations

(In one variable)

4.1 Introduction :

If x and y are two quantities; then both of these quantities will satisfy any one of the following four conditions (relations) :

i.e. either (i) $x > y$ (ii) $x \geq y$ (iii) $x < y$ or (iv) $x \leq y$

Each of the four conditions, given above, is an ***inequation***.

In the same way, each of the following also represents an inequation :

$x < 8$, $x \geq 5$, $-x + 4 \leq 3$, $x + 8 > 4$, etc.

4.2 Linear Inequations In One Variable :

If a, b and c are real numbers, then each of the following is called a **linear inequation** in one variable :

(i) $ax + b > c$. Read as : $ax + b$ is *greater than* c.

(ii) $ax + b < c$. Read as : $ax + b$ is *less than* c.

(iii) $ax + b \geq c$. Read as : $ax + b$ is *greater than* or *equal to* c.

(iv) $ax + b \leq c$. Read as : $ax + b$ is *less than* or *equal to* c.

In an inequation, the signs '>', '<', '≥' and '≤' are called signs of inequality.

4.3 Solving a Linear Inequation Algebrically :

To solve a given linear inequation means to find the value or values of the variable used in it.

Thus; (i) to solve the inequation $3x + 5 > 8$ means to find the variable x.

(ii) to solve the inequation $8 - 5y \leq 3$ means to find the variable y and so on

The following working rules must be adopted for solving a given linear inequation :

Rule 1 : *On transferring a positive term from one side of an inequation to its other side, the sign of the term becomes negative.*

e.g. $2x + 3 > 7 \Rightarrow 2x > 7 - 3$, $5x + 4 \leq 15 \Rightarrow 5x \leq 15 - 4$,

$23 \geq 4x + 15 \Rightarrow 23 - 15 \geq 4x$ and so on.

Rule 2 : *On transferring a negative term from one side of an inequation to its other side, the sign of the term becomes positive.*

e.g. $2x - 3 > 7 \Rightarrow 2x > 7 + 3$, $5x - 4 \leq 15 \Rightarrow 5x \leq 15 + 4$,

$23 \geq 4x - 15 \Rightarrow 23 + 15 \geq 4x$ and so on.

Rule 3 : *If each term of an inequation be multiplied or divided by the same positive number, the sign of inequality remains the same.*

That is, if p is positive

(i) $x < y \Rightarrow px < py$ and $\left(\frac{x}{p}\right) < \left(\frac{y}{p}\right)$,

(ii) $x > y \Rightarrow px > py$ and $\left(\frac{x}{p}\right) > \left(\frac{y}{p}\right)$,

(iii) $x \le y \Rightarrow px \le py$ and $\left(\frac{x}{p}\right) \le \left(\frac{y}{p}\right)$

and, (iv) $x \ge y \Rightarrow px \ge py$ and $\left(\frac{x}{p}\right) \ge \left(\frac{y}{p}\right)$.

Thus, $x \le 6 \Rightarrow 4x \le 4 \times 6$,

$x \ge 5 \Rightarrow 3x \ge 3 \times 5$,

$x \le 2 \Rightarrow \frac{x}{10} \le \frac{2}{10}$ and so on.

Rule 4 : *If each term of an inequation be multiplied or divided by the same negative number, the sign of inequality reverses.*

That is, if p is negative

(i) $x < y \Rightarrow px > py$ and $\left(\frac{x}{p}\right) > \left(\frac{y}{p}\right)$

(ii) $x \ge y \Rightarrow px \le py$ and $\left(\frac{x}{p}\right) \le \left(\frac{y}{p}\right)$

Thus, $x \le 6 \Rightarrow -4x \ge -4 \times 6$,

$x > 5 \Rightarrow -3x < -3 \times 5$

$x \ge y \Rightarrow \frac{x}{-2} \le \frac{y}{-2}$ and so on.

Rule 5 : *If sign of each term on both the sides of an inequation is changed, the sign of inequality gets reversed.*

i.e. (i) $-x > 5 \Leftrightarrow x < -5$ (ii) $3y \le 15 \Leftrightarrow -3y \ge -15$

(iii) $-2y < -7 \Leftrightarrow 2y > 7$ and so on.

Rule 6 : *If both the sides of an inequation are positive or both are negative, then on taking their reciprocals, the sign of inequality reverses.*

i.e. if x and y both are either positive or both are negative, then

(i) $x > y \Leftrightarrow \frac{1}{x} < \frac{1}{y}$ (ii) $x \le y \Leftrightarrow \frac{1}{x} \ge \frac{1}{y}$

(iii) $x \ge y \Leftrightarrow \frac{1}{x} \le \frac{1}{y}$ and so on.

4.4 Replacement Set and Solution Set :

The set, from which the value of the variable x is to be chosen, is called ***replacement set*** and its subset, whose elements satisfy the given inequation, is called ***solution set***.

e.g. Let the given inequation be $x < 3$, if :

(i) the replacement set = N, *the set of natural numbers*;
the solution set = {1, 2}

(ii) the replacement set = W, *the set of whole numbers*;
the solution set = {0, 1, 2}

(iii) the replacement set = Z or I, the *set of integers*;
the solution set = {..........., –2, –1, 0, 1, 2}

But, if the replacement set is the set of real numbers, the solution set can only be described in set-builder form, *i.e.* $\{x : x \in \text{R and } x < 3\}$.

1 **If the replacement set is the set of natural numbers (N), find the solution set of : (i) $3x + 4 < 16$ (ii) $8 - x \leq 4x - 2$.**

Solution :

(i) $3x + 4 < 16 \Rightarrow 3x < 16 - 4$ [Using rule 1]

$\Rightarrow 3x < 12$

$\Rightarrow \frac{3x}{3} < \frac{12}{3}$ [Using rule 3]

i.e. $x < 4$

Since, the replacement set = N (set of natural numbers)

∴ **Solution set = {1, 2, 3}** **Ans.**

(ii) $8 - x \leq 4x - 2 \Rightarrow -x - 4x \leq -2 - 8$ [Using rule 1]

$\Rightarrow -5x \leq -10$

$\Rightarrow \frac{-5x}{-5} \geq \frac{-10}{-5}$ [Using rule 4]

i.e. $x \geq 2$

Since, the replacement set = N

∴ **Solution set = {2, 3, 4, 5, 6, ...}**

Alternative method :

$8 - x \leq 4x - 2 \Rightarrow 4x - 2 \geq 8 - x$ [$x \leq y$ and $y \geq x$ mean the same]

$\Rightarrow 4x + x \geq 8 + 2$ [Using rule 2]

$\Rightarrow 5x \geq 10$

$\Rightarrow x \geq 2$ and $x \in$ N

∴ **Solution set = {2, 3, 4, 5, 6, ...}** **Ans.**

2 If the replacement set is the set of whole numbers (W), find the solution set of : (i) $5x + 4 \leq 24$ (ii) $4x - 2 < 2x + 10$.

Solution :

(i) $5x + 4 \leq 24 \Rightarrow 5x \leq 24 - 4$

$\Rightarrow 5x \leq 20$ and $x \leq \frac{20}{5}$ *i.e.* $x \leq 4$

Since, the replacement set is the set of whole numbers

$\therefore$ **Solution set = {0, 1, 2, 3, 4}** **Ans.**

(ii) $4x - 2 < 2x + 10 \Rightarrow 4x - 2x < 10 + 2$

$\Rightarrow 2x < 12$ and $x < 6$

Since, the replacement set = W(whole numbers)

$\therefore$ **Solution set = {0, 1, 2, 3, 4, 5}** **Ans.**

3 If the replacement set is the set of integers, (I or Z), between −6 and 8, find the solution set of : (i) $6x - 1 \geq 9 + x$ (ii) $15 - 3x > x - 3$.

Solution :

(i) $6x - 1 \geq 9 + x \Rightarrow 6x - x \geq 9 + 1$

$\Rightarrow 5x \geq 10$ and $x \geq 2$

Since, the replacement set is the set of integers between −6 and 8.

$\therefore$ **Solution set = {2, 3, 4, 5, 6, 7}** **Ans.**

(ii) $15 - 3x > x - 3 \Rightarrow -3x - x > -3 - 15$

$\Rightarrow -4x > -18$

$\Rightarrow \frac{-4x}{-4} < \frac{-18}{-4}$ [Using rule 4]

$\Rightarrow x < 4{\cdot}5$

Since, the replacement set is the set of integers between −6 and 8

$\therefore$ **Solution set = {−5, −4, −3, −2, −1, 0, 1, 2, 3, 4}** **Ans.**

Alternative method :

$15 - 3x > x - 3 \Rightarrow x - 3 < 15 - 3x$

$\Rightarrow x + 3x < 15 + 3$

$\Rightarrow 4x < 18$ and $x < 4{\cdot}5$

$\therefore$ **Solution set = {−5, −4, −3, −2, −1, 0, 1, 2, 3, 4}** **Ans.**

4 If the replacement set is the set of real numbers (R), find the solution set of : (i) $5 - 3x < 11$ (ii) $8 + 3x \geq 28 - 2x$.

Solution :

(i) $5 - 3x < 11 \Rightarrow -3x < 11 - 5$

$\Rightarrow \quad -3x < 6$

$\Rightarrow \quad \frac{-3x}{-3} > \frac{6}{-3}$ [Using rule 4]

$\Rightarrow \quad x > -2$

Since, the replacement set is the set of real numbers (R)

$\therefore$ **Solution set = $\{x : x > -2$ and $x \in R\}$** **Ans.**

(ii) **$8 + 3x \geq 28 - 2x$** $\Rightarrow \quad 3x + 2x \geq 28 - 8$

$\Rightarrow \quad 5x \geq 20$ and $x \geq 4$

Since, the replacement set is the set of real numbers

$\therefore$ **Solution set = $\{x : x \geq 4$ and $x \in R\}$** **Ans.**

5 Solve : $\frac{x}{2} - 5 \leq \frac{x}{3} - 4$, where x is a positive odd integer.

Solution :

$\frac{x}{2} - 5 \leq \frac{x}{3} - 4$ $\Rightarrow \quad \frac{x}{2} - \frac{x}{3} \leq -4 + 5$

$\Rightarrow \quad \frac{3x - 2x}{6} \leq 1 \quad \Rightarrow \quad x \leq 6$

Since, x is a positive odd integer

$\therefore$ **Solution set = {1, 3, 5}** **Ans.**

6 Solve the following inequation : $2y - 3 < y + 1 \leq 4y + 7$; if :
(i) $y \in$ {Integers} (ii) $y \in$ R (real numbers)

Solution :

$2y - 3 < y + 1 \leq 4y + 7$ $\Rightarrow \quad 2y - 3 < y + 1$ and $y + 1 \leq 4y + 7$

$\Rightarrow \quad y < 4$ and $-6 \leq 3y$

$\Rightarrow \quad y < 4$ and $y \geq -2$

$\Rightarrow \quad -2 \leq y < 4$

(i) **When $y \in$ {Integers}**

$\therefore$ **Solution set = {–2, –1, 0, 1, 2, 3}** **Ans.**

(ii) **When $y \in$ R (real numbers)**

$\therefore$ **Solution set = $\{y : -2 \leq y < 4$ and $y \in R\}$** **Ans.**

EXERCISE 4(A)

1. State, *true* or *false* :

   (i) $x < -y \Rightarrow -x > y$

   (ii) $-5x \geq 15 \Rightarrow x \geq -3$

   (iii) $2x \leq -7 \Rightarrow \frac{2x}{-4} \geq \frac{-7}{-4}$

   (iv) $7 > 5 \Rightarrow \frac{1}{7} < \frac{1}{5}$

2. State, whether the following statements are *true* or *false*.

   (i) If $a < b$, then $a - c < b - c$

   (ii) If $a > b$, then $a + c > b + c$

   (iii) If $a < b$, then $ac > bc$

   (iv) If $a > b$, then $\frac{a}{c} < \frac{b}{c}$

   (v) If $a - c > b - d$; then $a + d > b + c$

   (vi) If $a < b$, and $c > 0$, then $a - c > b - c$

   where a, b, c and d are real numbers and $c \neq 0$.

3. If $x \in$ N, find the solution set of inequations.

   (i) $5x + 3 \leq 2x + 18$

   (ii) $3x - 2 < 19 - 4x$

4. If the replacement set is the set of whole numbers, solve :

   (i) $x + 7 \leq 11$ (ii) $3x - 1 > 8$

   (iii) $8 - x > 5$ (iv) $7 - 3x \geq -\frac{1}{2}$

   (v) $x - \frac{3}{2} < \frac{3}{2} - x$ (vi) $18 \leq 3x - 2$

5. Solve the inequation :

   $3 - 2x \geq x - 12$ given that $x \in$ N.

6. If $25 - 4x \leq 16$, find :

   (i) the smallest value of x, when x is a real number,

   (ii) the smallest value of x, when x is an integer.

7. If the replacement set is the set of real numbers, solve :

   (i) $-4x \geq -16$ (ii) $8 - 3x \leq 20$

   (iii) $5 + \frac{x}{4} > \frac{x}{5} + 9$ (iv) $\frac{x+3}{8} < \frac{x-3}{5}$

8. Find the smallest value of x for which $5 - 2x < 5\frac{1}{2} - \frac{5}{3}x$, where x is an integer.

9. Find the largest value of x for which $2(x - 1) \leq 9 - x$ and $x \in$ W.

10. Solve the inequation :

    $12 + 1\frac{5}{6}x \leq 5 + 3x$ and $x \in$ R.

11. Given $x \in$ {integers}, find the solution set of : $-5 \leq 2x - 3 < x + 2$.

12. Given x $\in$ {whole numbers}, find the solution set of : $-1 \leq 3 + 4x < 23$.

4.5 Representation of the solution on the number line :

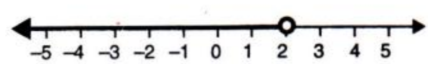
A real number line can be used to represent the solution set of an inequation.

The convention is that o (a hollow circle) marks the end of a range with a *strict inequality* (*i.e.* < or >) and • (a darkened circle) marks the end of a range involving an equality as well (*i.e.* ≤ or ≥).

For Example :

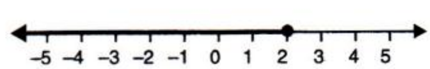
The adjacent figure shows

$x < 2$ and $x \in$ R.

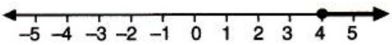


The number 2 is encircled and the circle is not darkened to show that 2 is not included in the graph.

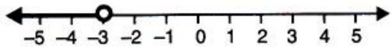
If 2 is also included *i.e.* $x \leq 2$, then the circle will be darkened and the graph will be as shown alongside:



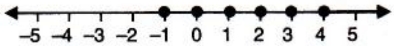
Similarly, study each of the following carefully :

(i) 

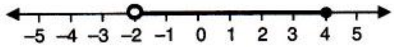
$\{x : x \geq 4 \text{ and } x \in R\}$

(ii) 

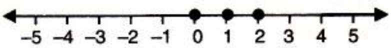
$\{x : x < -3 \text{ and } x \in R\}$

(iii) 

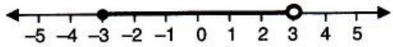
$\{x : -2 < x \leq 4 \text{ and } x \in Z\}$

(iv) 

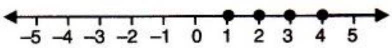
$\{x : -2 < x \leq 4 \text{ and } x \in R\}$

(v) 

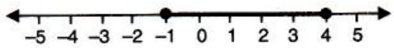
$\{x : -3 \leq x < 3 \wedge x \in W\}$

(vi) 

$\{x : -3 \leq x < 3 \text{ and } x \in R\}$

(vii) 

$\{x : -1 \leq x \leq 4 \wedge x \in N\}$

(viii) 

$\{x : -1 \leq x \leq 4 \wedge x \in R\}$

[Note: The symbol 'Λ' stands for 'and'].

7 Given that $x \in R$, solve the following inequality and graph the solution on the number line : $-1 \leq 3 + 4x < 23$ [2006]

Solution :

Given : $-1 \leq 3 + 4x < 23;\ x \in R$

$\Rightarrow \quad -1 \leq 3 + 4x \quad \text{and} \quad 3 + 4x < 23$

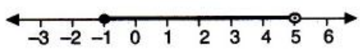
$\Rightarrow \quad -4 \leq 4x \quad \text{and} \quad 4x < 20$

$\Rightarrow \quad -1 \leq x \quad \text{and} \quad x < 5$

$\Rightarrow \quad -1 \leq x < 5;\ x \in R$

$\therefore$ **Solution = $\{-1 \leq x < 5 : x \in R\}$** **Ans.**

Solution on the number line is :

= 

Ans.

8 Simplify : $-\frac{1}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$: $x \in R$.

Graph the values of x on the real number line.

Solution :

The given inequation has two parts :

$-\frac{1}{3} \leq \frac{x}{2} - 1\frac{1}{3}$ and $\frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$

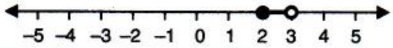
$\Rightarrow \quad -\frac{1}{3} + 1\frac{1}{3} \leq \frac{x}{2}$ | $\Rightarrow \quad \frac{x}{2} < \frac{1}{6} + \frac{4}{3}$

$\Rightarrow \quad 1 \leq \frac{x}{2}$ | $\Rightarrow \quad \frac{x}{2} < \frac{9}{6}$

$\Rightarrow \quad 2 \leq x$ | $\Rightarrow \quad x < \frac{9}{6} \times 2$

| $\Rightarrow \quad x < 3$

∴ **On simplifying,** the given inequation reduces to $2 \le x < 3$ and **the required graph on number line is :** **Ans.**



9 List the solution set of $50 - 3(2x - 5) < 25$, given that $x \in W$.

Also, represent the solution set obtained on a number line.

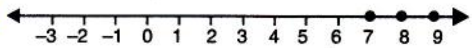
Solution :

$$50 - 3(2x - 5) < 25 \Rightarrow 50 - 6x + 15 < 25$$
$$\Rightarrow -6x < 25 - 65$$
$$\Rightarrow -6x < -40$$
$$\Rightarrow \frac{-6x}{-6} > \frac{-40}{-6}$$

Division by a negative number reverses the sign of inequality.

$$\Rightarrow x > 6\frac{2}{3}$$

Required solution set = {7, 8, 9, ... } **Ans.**

And, **the required number line is :**  **Ans.**

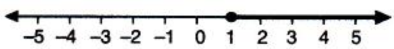
4.6 Combining Inequations :

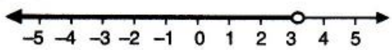
10 Solve and graph the solution set of $3x + 6 \ge 9$ and $-5x > -15$; where $x \in R$.

Solution :

$$3x + 6 \ge 9 \Rightarrow 3x \ge 9 - 6$$
$$\Rightarrow 3x \ge 3 \qquad \Rightarrow \mathbf{x \ge 1}$$

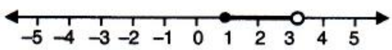
And, $-5x > -15 \Rightarrow \frac{-5x}{-5} < \frac{-15}{-5} \Rightarrow \mathbf{x < 3}$

Graph for $x \ge 1$: 

Graph for $x < 3$: 

∴ **Graph of solution set of $x \ge 1$ and $x < 3$**

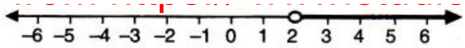
= Graph of points common to both $x \ge 1$ and $x < 3$

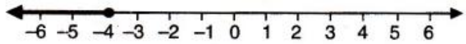
=  **Ans.**

11 Solve and graph the solution set of $-2 < 2x - 6$ or $-2x + 5 \ge 13$; where $x \in R$.

Solution :

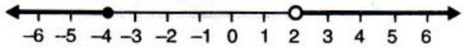
$$-2 < 2x - 6 \Rightarrow 2x - 6 > -2$$
$$\Rightarrow 2x > 4 \qquad \Rightarrow \mathbf{x > 2}$$
$$-2x + 5 \ge 13 \Rightarrow -2x \ge 8$$
$$\Rightarrow \mathbf{x \le -4}$$

Graph for $x > 2$: 

Graph for $x \leq -4$: 

$\therefore$ **Graph of solution set of $x > 2$ or $x \leq -4$**

= Graph of points which belong to $x > 2$ or $x \leq -4$ or both

= 

Ans.

12 Given : $P = \{x : 5 < 2x - 1 \leq 11, x \in R\}$
$Q = \{x : -1 \leq 3 + 4x < 23, x \in I\}$
where R = {real numbers} and I = {integers}.
Represent P and Q on two different number lines. Write down the elements of $P \cap Q$.

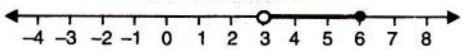
Solution :

For P : $5 < 2x - 1 \leq 11$, where $x \in R$

$\Rightarrow$ $5 < 2x - 1$ and $2x - 1 \leq 11$

$\Rightarrow$ $3 < x$ and $x \leq 6$

i.e. **$3 < x \leq 6$; $x \in R$**

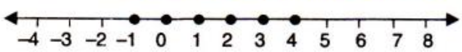
$\therefore$ **P** = 

For Q : $-1 \leq 3 + 4x < 23$, $x \in I$

$\Rightarrow$ $-1 \leq 3 + 4x$ and $3 + 4x < 23$

$\Rightarrow$ $-1 \leq x$ and $x < 5$

i.e. **$-1 \leq x < 5$; where $x \in I$**

$\therefore$ **Q** = 

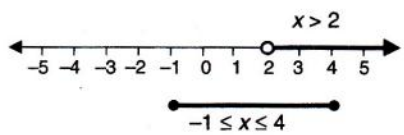
Hence, **$P \cap Q$** = {elements common to both P and Q} = **{4}.** **Ans.**

13 Write down the range of values of x ($x \in R$) for which both the inequations $x > 2$ and $-1 \leq x \leq 4$ are true.

Solution :

Both the given inequations are true in the range; where their graphs on the real number lines overlap.

The graphs for the given inequalities are drawn alongside :



It is clear from both the graphs that their common range is $2 < x \leq 4$.

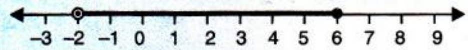
$\therefore$ **Required range is $2 < x \leq 4$** **Ans.**

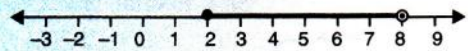
For any two solution sets A and B :

(i) A and B = Intersection of sets A and B
= Set of elements common to set A and to set B
= $A \cap B$

(ii) A or B = Union of sets A and B
= Set of elements which belong either set A and to set B
= $A \cup B$

14 The diagram, given below, represents two inequations P and Q on real number lines :

P = 

Q = 

(i) Write down P and Q in set builder notation.

(ii) Represent each of the following sets on different number lines :

(a) $P \cup Q$ (b) $P \cap Q$ (c) $P - Q$

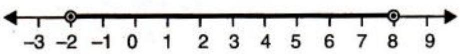
(d) $Q - P$ (e) $P \cap Q'$ (f) $P' \cap Q$

Solution :

(i) $\mathbf{P = \{x : -2 < x \leq 6 \text{ and } x \in R\}}$ and,

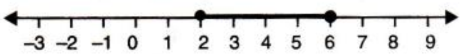
$\mathbf{Q = \{x : 2 \leq x < 8 \text{ and } x \in R\}}$ **Ans.**

(ii) (a) $\mathbf{P \cup Q}$ = Numbers which belong to P or to Q or to both

= 

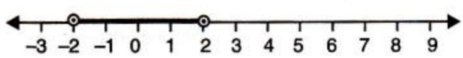
Ans.

(b) $\mathbf{P \cap Q}$ = Numbers common to both P and Q

= 

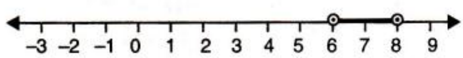
Ans.

(c) $\mathbf{P - Q}$ = Numbers which belong to P but do not belong to Q

= 

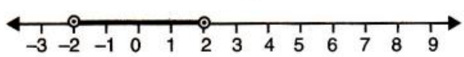
Ans.

(d) $\mathbf{Q - P}$ = Numbers which belong to Q but do not belong to P

= 

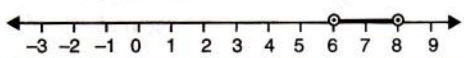
Ans.

(e) $\mathbf{P \cap Q'}$ = Numbers which belong to P but do not belong to Q = P – Q

= 

Ans.

(f) $\mathbf{P' \cap Q}$ = Numbers which do not belong to P but belong to Q = Q – P

= 

Ans.

15 Find three smallest consecutive whole numbers such that the difference between one-fourth of the largest and one-fifth of the smallest is at least 3.

Solution :

Let the required whole numbers be x, $x + 1$ and $x + 2$.

According to the given statement :

$$\frac{x+2}{4} - \frac{x}{5} \geq 3 \Rightarrow \frac{5x+10-4x}{20} \geq 3$$

$$\Rightarrow x + 10 \geq 60 \text{ i.e., } x \geq 50$$

Since, the smallest value of $x = 50$ that satisfies the inequation $x \geq 50$.

∴ **Required smallest consecutive whole numbers are :**

$= x, x + 1$ and $x + 2$

$= 50, 50 + 1$ and $50 + 2$

$=$ **50, 51 and 52** **Ans.**

EXERCISE 4(B)

1. Represent the following inequalities on real number lines :

   (i) $2x - 1 < 5$ (ii) $3x + 1 \geq - 5$

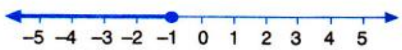
   (iii) $2\,(2x - 3) \leq 6$ (iv) $-4 < x < 4$

   (v) $-2 \leq x < 5$ (vi) $8 \geq x > - 3$

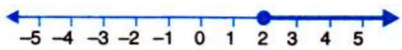
   (vii) $-5 < x \leq -1$

2. For each graph given alongside, write an inequation taking x as the variable :

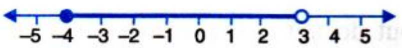
   (i)



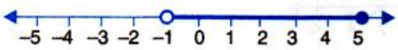
   (ii)



   (iii)



   (iv)



3. For the following inequations, graph the solution set on the real number line :

   (i) $-4 \leq 3x - 1 < 8$

   (ii) $x - 1 < 3 - x \leq 5$

4. Represent the solution of each of the following inequalities on the real number line:

   (i) $4x - 1 > x + 11$

   (ii) $7 - x \leq 2 - 6x$

   (iii) $x + 3 \leq 2x + 9$

   (iv) $2 - 3x > 7 - 5x$

   (v) $1 + x \geq 5x - 11$

   (vi) $\dfrac{2x+5}{3} > 3x - 3$

5. $x \in$ {real numbers} and $-1 < 3 - 2x \leq 7$, evaluate x and represent it on a number line.

6. List the elements of the solution set of the inequation $-3 < x - 2 \leq 9 - 2x;\ x \in N$.

7. Find the range of values of x which satisfies

   $$-2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3},\ x \in R.$$

   Graph these values of x on the number line. **[2007]**

8. Find the values of x, which satisfy the inequation:

   $$-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6},\ x \in N.$$

   Graph the solution on the number line.

9. Given $x \in$ {real numbers}, find the range of values of x for which $-5 \leq 2x - 3 < x + 2$ and represent it on a real number line.

10. If $5x - 3 \leq 5 + 3x \leq 4x + 2$, express it as $a \leq x \leq b$ and then state the values of a and b.

11. Solve the following inequation and graph the solution set on the number line :

    $2x - 3 < x + 2 \leq 3x + 5;\ x \in R.$

12. Solve and graph the solution set of :

    (i) $2x - 9 < 7$ and $3x + 9 \leq 25;\ x \in R.$

    (ii) $2x - 9 \leq 7$ and $3x + 9 > 25;\ x \in I.$

    (iii) $x + 5 \geq 4\,(x - 1)$ and $3 - 2x < -7;\ x \in R.$

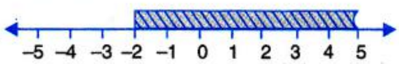
13. Solve and graph the solution set of :

    (i) $3x - 2 > 19$ or $3 - 2x \geq - 7;\ x \in R.$

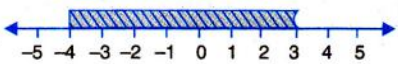
    (ii) $5 > p - 1 > 2$ or $7 \leq 2p - 1 \leq 17;\ p \in R.$

14. The diagram represents two inequations A and B on real number lines :

    A =



    B =



    (i) Write down A and B in set builder notation.

    (ii) Represent $A \cap B$ and $A \cap B'$ on two different number lines.

15. Use real number line to find the range of values of x for which :

    (i) $x > 3$ and $0 < x < 6$.

    (ii) $x < 0$ and $- 3 \leq x < 1$.

    (iii) $-1 < x \leq 6$ and $-2 \leq x \leq 3$

16. Illustrate the set $\{x : -3 \leq x < 0 \text{ or } x > 2; x \in R\}$ on a real number line.

17. Given $A = \{x : -1 < x \leq 5, x \in R\}$ and $B = \{x : -4 \leq x < 3, x \in R\}$

Represent on different number lines :

(i) $A \cap B$ (ii) $A' \cap B$ (iii) $A - B$

18. P is the solution set of $7x - 2 > 4x + 1$ and Q is the solution set of $9x - 45 \geq 5(x - 5)$; where $x \in R$. Represent :

(i) $P \cap Q$

(ii) $P - Q$

(iii) $P \cap Q'$ on different number lines.

19. If $P = \{x : 7x - 4 > 5x + 2, x \in R\}$ and $Q = \{x : x - 19 \geq 1 - 3x, x \in R\}$; find the range of set $P \cap Q$ and represent it on a number line.

20. Find the range of values of x, which satisfy:

$$-\frac{1}{3} \leq \frac{x}{2} + 1\frac{2}{3} < 5\frac{1}{6}$$

Graph, in each of the following cases, the values of x on the different real number lines:

(i) $x \in W$ (ii) $x \in Z$ (iii) $x \in R$.

21. Given : $A = \{x : -8 < 5x + 2 \leq 17, x \in I\}$

$B = \{x : -2 \leq 7 + 3x < 17, x \in R\}$

Where R = {real numbers} and I = {integers}.

Represent A and B on two different number lines. Write down the elements of $A \cap B$.

22. Solve the following inequation and represent the solution set on the number line $2x - 5 \leq 5x + 4 < 11$, where $x \in I$. **[2011]**

23. Given that $x \in I$, solve the inequation and graph the solution on the number line :

$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2.$$ **[2004]**

24. Given :

$A = \{x : 11x - 5 > 7x + 3, x \in R\}$ and

$B = \{x : 18x - 9 \geq 15 + 12x, x \in R\}$.

Find the range of set $A \cap B$ and represent it on a number line **[2005]**

25. Find the set of values of x, satisfying :

$7x + 3 \geq 3x - 5$ and $\frac{x}{4} - 5 \leq \frac{5}{4} - x$,

where $x \in N$.

26. Solve :

(i) $\frac{x}{2} + 5 \leq \frac{x}{3} + 6$, where x is a positive odd integer

(ii) $\frac{2x+3}{3} \geq \frac{3x-1}{4}$, where x is a positive even integer

27. Solve the inequation :

$$-2\frac{1}{2} + 2x \leq \frac{4x}{5} \leq \frac{4}{3} + 2x, x \in W.$$

Graph the solution set on the number line.

28. Find three consecutive largest positive integers such that the sum of one-third of first, one-fourth of second and one-fifth of third is at most 20.

29. Solve the given inequation and graph the solution on the number line.

$2y - 3 < y + 1 \leq 4y + 7, y \in R$ **[2008]**

30. Solve the inequation :

$3z - 5 \leq z + 3 < 5z - 9; z \in R$.

Graph the solution set on the number line.

31. Solve the following inequation and represent the solution set on the number line.

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in R.$$ **[2010]**

32. Solve the following inequation and represent the solution set on the number line :

$$4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x, x \in R$$ **[2012]**

33. Solve the following inequation, write the solution set and represent it on the number line:

$$-\frac{x}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$$ **[2013]**

34. Find the values of x, which satisfy the inequation

$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2, x \in W$. Graph the solution set on the number line. **[2014]**

35. Solve the following inequation and write the solution set :

$13x - 5 < 15x + 4 < 7x + 12, x \in R$

Represent the solution on a real number line. **[2015]**

4.7 Important :

1. If the product of two numbers is less than 0 (zero) *i.e.* product is negative; one of the two numbers is positive and the other is negative.
2. If the product of two numbers is greater than 0 (zero) *i.e.* positive; both the numbers are positive or both are negative.

1. For $(x - 3)(x + 5) < 0$

If $(x - 3)$ is positive, then $x + 5$ is negative

and if $x - 3$ is negative, then $x + 5$ is positive

Case 1 : When $(x - 3)$ is positive and $(x + 5)$ is negative

i.e. $x - 3 > 0$ and $x + 5 < 0$

$\Rightarrow$ $x > 3$ and $x < -5$

It is not possible, as number greater than 3 cannot be smaller than -5.

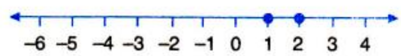
Case 2 : When $(x - 3)$ is negative and $(x + 5)$ is positive

i.e. $x - 3 < 0$ and $x + 5 > 0$

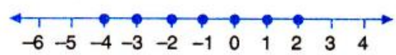
$\Rightarrow$ $x < 3$ and $x > -5$

$\Rightarrow$ $-5 < x < 3$, **which is possible.**

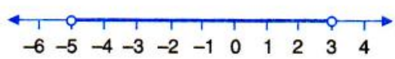
If $x \in N$, then : $-5 < x < 3$ on the number line is :



If $x \in Z$, then : $-5 < x < 3$ on the number line is :



If $x \in R$, then : $-5 < x < 3$ on the number line is :



2. For $(x - 3)(x + 5) > 0$

If $(x - 3)$ is negative, then $(x + 5)$ is also negative

and, if $(x - 3)$ is positive, then $(x + 5)$ is also positive

Case 1 : $x - 3$ is negative and $x + 5$ is negative

i.e. $x - 3 < 0$ and $x + 5 < 0$

$\Rightarrow$ $x < 3$ and $x < -5$

$\Rightarrow$ $x < -5$ as the number smaller than -5 is less than 3 also.

Case 2 : $x - 3$ is positive and $x + 5$ is positive

i.e. $x - 3 > 0$ and $x + 5 > 0$

$\Rightarrow$ $x > 3$ and $x > -5$

$\Rightarrow$ $x > 3$ as the number greater than 3 is also greater than -5.

If $x \in R$, then for $(x - 3)(x + 5) > 0$ the number line is :