5 Quadratic Equations

5.1 Introduction :

An equation with one variable, in which the **highest power** of the variable is **two**, is known as ***quadratic equation***.

For example :

(i) $3x^2 + 4x + 7 = 0$ (iii) $2x^2 - 50 = 0$

(ii) $4x^2 + 5x = 0$ (iv) $x^2 = 4$, etc.

1. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are all real numbers and $a \neq 0$.

   e.g. equation $4x^2 + 5x - 6 = 0$ is a quadratic equation in standard form.

2. Every quadratic equation gives two values of the unknown variable used in it and these values are called ***roots of the equation.***

3. **Discriminant :** For the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$; the expression $\boldsymbol{b^2 - 4ac}$ is called discriminant and is, in general, denoted by the letter '$\boldsymbol{D}$'.

   Thus, discriminant $\boldsymbol{D = b^2 - 4ac}$.

4. If a quadratic equation contains only two terms one square term and one first power term of the unknown, it is called ***adfected quadratic equation***.

   For example : (i) $4x^2 + 5x = 0$ (ii) $7x^2 - 3x = 0$, etc.

5. If the quadratic equation contains only the square of the unknown, it is called ***pure quadratic equation***.

   For example : (i) $x^2 = 4$ (ii) $3x^2 - 8 = 0$, etc.

5.2 To examine the nature of the roots :

Examining the roots of a quadratic equation means to know the type of its roots *i.e.* whether they are *real* or *imaginary*, *rational* or *irrational*, *equal* or *unequal*.

The nature of the roots of a quadratic equation depends entirely on the value of its discriminant $b^2 - 4ac$.

If for a quadratic equation $ax^2 + bx + c = 0$; where a, b and c are real numbers and $a \neq 0$, then discriminant :

(i) $\boldsymbol{b^2 - 4ac = 0}$ $\Rightarrow$ the roots are **real** and **equal**.

(ii) $\boldsymbol{b^2 - 4ac > 0}$ $\Rightarrow$ the roots are **real** and **unequal**.

(iii) $\boldsymbol{b^2 - 4ac < 0}$ $\Rightarrow$ the roots are **imaginary** (not real).

1. Every number, whether it is rational or irrational, is a real number. *i.e.*
   (i) every rational number is a real number and
   (ii) every irrational number is also a real number.
2. Square root of a negative number is an imaginary number.
   Thus : each of $\sqrt{-4}$, $\sqrt{-8}$, $2\sqrt{-5}$, ..., etc. is an imaginary number.

1 Without solving, examine the nature of the roots of the equations :

(i) $5x^2 - 6x + 7 = 0$ (ii) $x^2 + 6x + 9 = 0$ (iii) $2x^2 + 6x + 3 = 0$

Solution :

(i) Comparing given quadratic equation $5x^2 - 6x + 7 = 0$ with equation $ax^2 + bx + c = 0$; we get : $a = 5$, $b = -6$ and $c = 7$.

$\Rightarrow$ Discriminant $= b^2 - 4ac = (-6)^2 - 4 \times 5 \times 7$

$= 36 - 140 = -104$; ***which is negative.***

Since, a, b and c are real numbers; $a \neq 0$ and $b^2 - 4ac < 0$.

$\therefore$ **The roots are not real *i.e.* the roots are imaginary.** **Ans.**

(ii) Comparing quadratic equation $x^2 + 6x + 9 = 0$ with $ax^2 + bx + c = 0$; we get : $a = 1$, $b = 6$ and $c = 9$

$\Rightarrow \quad b^2 - 4ac = (6)^2 - 4 \times 1 \times 9 = 36 - 36 = 0$

Since; a, b and c are real numbers; $a \neq 0$ and $b^2 - 4ac = 0$.

$\therefore$ **The roots are real and equal.** **Ans.**

(iii) Comparing $2x^2 + 6x + 3 = 0$ and $ax^2 + bx + c$, we get : $a = 2$, $b = 6$ and $c = 3$

$b^2 - 4ac = (6)^2 - 4 \times 2 \times 3$

$= 36 - 24 = 12$; ***which is positive.***

Since; a, b and c are real numbers; $a \neq 0$ and $b^2 - 4ac > 0$.

$\therefore$ **The roots are real and unequal.** **Ans.**

2 Find the value of 'm', if the roots of the following quadratic equation are equal: $(4 + m)x^2 + (m + 1)x + 1 = 0$.

Solution :

For the given equation $(4 + m)x^2 + (m + 1)x + 1 = 0$;

$a = 4 + m$, $b = m + 1$ and $c = 1$

Since, the roots are equal

$\therefore \quad b^2 - 4ac = 0 \Rightarrow (m + 1)^2 - 4(4 + m) \times 1 = 0$

$\Rightarrow \quad m^2 + 2m + 1 - 16 - 4m = 0$

$\Rightarrow \quad m^2 - 2m - 15 = 0$

On solving, we get : $m = 5$ or $m = -3$ **Ans.**

EXERCISE 5(A)

1. Without solving, comment upon the nature of roots of each of the following equations :
   (i) $7x^2 - 9x + 2 = 0$ (ii) $6x^2 - 13x + 4 = 0$
   (iii) $25x^2 - 10x + 1 = 0$ (iv) $x^2 + 2\sqrt{3}x - 9 = 0$
   (v) $x^2 - ax - b^2 = 0$ (vi) $2x^2 + 8x + 9 = 0$
2. Find the value of 'p', if the following quadratic equations have equal roots :
   (i) $4x^2 - (p - 2)x + 1 = 0$
   (ii) $x^2 + (p - 3)x + p = 0$ **[2013]**
3. The equation $3x^2 - 12x + (n - 5) = 0$ has equal roots. Find the value of n.
4. Find the value of 'm', if the following equation has equal roots :
   $$(m - 2)x^2 - (5 + m)x + 16 = 0$$
5. Find the value of k for which the equation $3x^2 - 6x + k = 0$ has distinct and real root. **[2015]**

5.3 Solving quadratic equations by factorisation :

Steps : (i) Clear all fractions and brackets, if necessary.
(ii) Transpose all the terms to the left hand side to get an equation in the form $ax^2 + bx + c = 0$.
(iii) Factorise the expression on the left hand side.
(iv) Put each factor equal to zero and solve.

Zero Product Rule : Whenever the product of two expressions is zero; at least one of the expressions is zero.

Thus, if $(x + 3)(x - 2) = 0$
$\Rightarrow x + 3 = 0$, or $x - 2 = 0$
$\Rightarrow x = -3$, or $x = 2$.

3 Solve : (i) $2x^2 - 7x = 39$ (ii) $x^2 = 5x$ (iii) $x^2 = 16$

Solution :

(i) $2x^2 - 7x = 39$
$\Rightarrow 2x^2 - 7x - 39 = 0$ [Expressing as $ax^2 + bx + c = 0$]
$\Rightarrow 2x^2 - 13x + 6x - 39 = 0$ [Factorising the left hand side]
$\Rightarrow x(2x - 13) + 3(2x - 13) = 0$
$(2x - 13)(x + 3) = 0$
$2x - 13 = 0$, or $x + 3 = 0$ [Zero Product Rule]
$\Rightarrow x = \frac{13}{2}$, or $x = -3$ **Ans.**

(ii) $x^2 = 5x \Rightarrow x^2 - 5x = 0$
$\Rightarrow x(x - 5) = 0$
$\Rightarrow x = 0$, or $x - 5 = 0$
$\Rightarrow x = 0$, or $x = 5$ **Ans.**

(iii) $x^2 = 16 \Rightarrow x^2 - 16 = 0$
$\Rightarrow (x + 4)(x - 4) = 0$
$\Rightarrow x + 4 = 0$, or $x - 4 = 0$
$\Rightarrow x = -4$, or $x = 4$ **Ans.**

Alternative method :
$x^2 = 16$
$\Rightarrow x = \pm 4$
$\Rightarrow x = 4$ or $x = -4$ **Ans.**

4 Solve : $\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}$.

Solution :

$$\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}$$

$$\Rightarrow \quad \frac{x^2 + (x-1)^2}{x(x-1)} = \frac{5}{2}$$

$$2(x^2 + x^2 - 2x + 1) = 5(x^2 - x)$$

$$\Rightarrow \quad 4x^2 - 4x + 2 = 5x^2 - 5x$$

$$\Rightarrow \quad -x^2 + x + 2 = 0$$

$$\Rightarrow \quad x^2 - x - 2 = 0 \quad \text{[Changing the sign of each term]}$$

$$\Rightarrow \quad (x-2)(x+1) = 0 \quad \text{[On factorising]}$$

$$\Rightarrow \quad x - 2 = 0, \text{ or } x + 1 = 0 \quad \text{[Zero Product Rule]}$$

$$\Rightarrow \quad x = 2, \text{ or } x = -1$$

Ans.

5 Find the quadratic equation whose solution set is {–2, 3}.

Solution :

Since, solution set = {–2, 3}

$\Rightarrow$ Roots are –2 and 3

$$\Rightarrow \quad x = -2, \text{ or } x = 3$$

$$\Rightarrow \quad x + 2 = 0, \text{ or } x - 3 = 0$$

$$\Rightarrow \quad (x+2)(x-3) = 0$$

$$\Rightarrow \quad x^2 - 3x + 2x - 6 = 0$$

$\Rightarrow$ $\boldsymbol{x^2 - x - 6 = 0}$***; which is the required quadratic equation.*** **Ans.**

6 Use the substitution $x = 3y + 1$ to solve for y, if $5(3y + 1)^2 + 6(3y + 1) - 8 = 0$.

Solution :

$$5(3y+1)^2 + 6(3y+1) - 8 = 0$$

$$\Rightarrow \quad 5x^2 + 6x - 8 = 0 \quad \text{[Putting } 3y + 1 = x\text{]}$$

$$\Rightarrow \quad (x+2)(5x-4) = 0 \quad \text{[On factorising]}$$

$$\Rightarrow \quad x = -2, \text{ or } x = \frac{4}{5}$$

When $x = -2 \Rightarrow 3y + 1 = -2 \Rightarrow y = -1$

and, when $x = \frac{4}{5} \Rightarrow 3y + 1 = \frac{4}{5} \Rightarrow y = -\frac{1}{15}$

$$\therefore y = -1, \text{ or } y = -\frac{1}{15}$$

Ans.

7 Without solving the quadratic equation $3x^2 - 2x - 1 = 0$, find whether $x = 1$ is a solution (root) of this equation or not.

Solution :

Substituting $x = 1$ in the given equation $3x^2 - 2x - 1 = 0$,

we get : $3(1)^2 - 2 \times 1 - 1 = 0$

$\Rightarrow$ $3 - 2 - 1 = 0$; *which is true.*

$\therefore$ **$x = 1$ is a solution of the given equation $3x^2 - 2x - 1 = 0$** **Ans.**

8 Without solving equation $x^2 - x + 1 = 0$; find whether $x = -1$ is a root of this equation or not.

Solution :

Substituting $x = -1$ in the given equation $x^2 - x + 1 = 0$,

we get : $(-1)^2 - (-1) + 1 = 0$

i.e. $1 + 1 + 1 = 0 \Rightarrow 3 = 0$; *which is not true.*

$\therefore$ **$x = -1$ is not a root of the given equation $x^2 - x + 1 = 0$** **Ans.**

9 Find the value of k for which $x = 2$ is a root (solution) of equation $kx^2 + 2x - 3 = 0$.

Solution :

Substituting $x = 2$ in the given equation $kx^2 + 2x - 3 = 0$; we get :

$k(2)^2 + 2 \times 2 - 3 = 0$

$\Rightarrow$ $4k + 4 - 3 = 0 \Rightarrow k = -\frac{1}{4}$ **Ans.**

10 If $x = 2$ and $x = 3$ are roots of the equation $3x^2 - 2mx + 2n = 0$; find the values of m and n.

Solution :

$x = 2$ is a root of the equation $3x^2 - 2mx + 2n = 0$

$\Rightarrow$ $3(2)^2 - 2m \times 2 + 2n = 0$

$\Rightarrow$ $12 - 4m + 2n = 0$

$\Rightarrow$ $-4m + 2n = -12$ *i.e.* $2m - n = 6$ I

$x = 3$ is a root of the equation $3x^2 - 2mx + 2n = 0$

$\Rightarrow$ $3(3)^2 - 2m \times 3 + 2n = 0$

$\Rightarrow$ $27 - 6m + 2n = 0$

$\Rightarrow$ $-6m + 2n = -27$ *i.e.* $6m - 2n = 27$ II

On solving equations I and II, we get :

$m = 7{\cdot}5$ and **$n = 9$** **Ans.**

11 If one root of the quadratic equation $2x^2 + ax - 6 = 0$ is 2, find the value of a. Also, find the other root.

Solution :

Since, $x = 2$ is a root of the given equation $2x^2 + ax - 6 = 0$

$\Rightarrow \quad 2(2)^2 + a \times 2 - 6 = 0$ *i.e.* $8 + 2a - 6 = 0$ and $a = -1$ **Ans.**

Substituting $a = -1$, we get :

$2x^2 + (-1)x - 6 = 0 \quad [\because 2x^2 + ax - 6 = 0]$

$\Rightarrow \quad 2x^2 - x - 6 = 0$

$\Rightarrow \quad 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow \quad 2x(x - 2) + 3(x - 2) = 0$

$\Rightarrow \quad (x - 2)(2x + 3) = 0 \quad \Rightarrow x = 2$ or $x = \dfrac{-3}{2}$

$\Rightarrow$ **The other root** $= \dfrac{-3}{2}$ **Ans.**

12 Find the value of 'K' for which $x = 3$ is a solution of the quadratic equation, $(K + 2)x^2 - Kx + 6 = 0$

Hence, find the other root of the equation. **[2015]**

Solution :

$x = 3$ is a solution of equation $(K + 2)x^2 - Kx + 6 = 0$

$\Rightarrow \quad (K + 2) \times 9 - K \times 3 + 6 = 0$

$\Rightarrow \quad 9K + 18 - 3K + 6 = 0$ *i.e.* $6K = -24$ and $K = -4$ **Ans.**

For $K = -4$, $(K + 2)x^2 - Kx + 6 = 0$

$\Rightarrow \quad -2x^2 + 4x + 6 = 0$ *i.e.* $x^2 - 2x - 3 = 0$

$\Rightarrow \quad x^2 - 3x + x - 3 = 0$ *i.e.* $x(x - 3) + 1(x - 3) = 0$

$\Rightarrow \quad (x - 3)(x + 1) = 0$ *i.e.* $x = 3$ or $x = -1$

Since, $x = 3$ is already given to be one root (solution) of the equation.

$\therefore$ The other root of the equation is $x = -1$. **Ans.**

EXERCISE 5(B)

Solve equations, number 1 to number 20, given below, using factorisation method :

1. $x^2 - 10x - 24 = 0$
2. $x^2 - 16 = 0$
3. $2x^2 - \frac{1}{2}x = 0$
4. $x(x - 5) = 24$
5. $\frac{9}{2}x = 5 + x^2$
6. $\frac{6}{x} = 1 + x$
7. $x = \frac{3x + 1}{4x}$
8. $x + \frac{1}{x} = 2{\cdot}5$
9. $(2x - 3)^2 = 49$
10. $2(x^2 - 6) = 3(x - 4)$
11. $(x + 1)(2x + 8) = (x + 7)(x + 3)$

12. $x^2 - (a + b)x + ab = 0$

13. $(x + 3)^2 - 4(x + 3) - 5 = 0$

14. $4(2x - 3)^2 - (2x - 3) - 14 = 0$

15. $\frac{3x-2}{2x-3} = \frac{3x-8}{x+4}$

16. $2x^2 - 9x + 10 = 0$, when :

(i) $x \in N$ (ii) $x \in Q$.

17. $\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$

18. $\frac{4}{x+2} - \frac{1}{x+3} = \frac{4}{2x+1}$

19. $\frac{5}{x-2} - \frac{3}{x+6} = \frac{4}{x}$

20. $\left(1+\frac{1}{x+1}\right)\left(1-\frac{1}{x-1}\right) = \frac{7}{8}$

21. Find the quadratic equation, whose solution set is :

(i) $\{3, 5\}$ (ii) $\{-2, 3\}$

22. (i) Solve : $\frac{x}{3} + \frac{3}{6-x} = \frac{2(6+x)}{15}$; $(x \neq 6)$

(ii) Solve the equation $9x^2 + \frac{3x}{4} + 2 = 0$, if possible, for real values of x.

23. Find the value of x, if $a + 1 = 0$ and $x^2 + ax - 6 = 0$.

24. Find the value of x, if $a + 7 = 0$; $b + 10 = 0$ and $12x^2 = ax - b$.

25. Use the substitution $y = 2x + 3$ to solve for x, if $4(2x + 3)^2 - (2x + 3) - 14 = 0$.

26. Without solving the quadratic equation $6x^2 - x - 2 = 0$, find whether $x = \frac{2}{3}$ is a solution of this equation or not.

27. Determine whether $x = -1$ is a root of the equation $x^2 - 3x + 2 = 0$ or not.

28. If $x = \frac{2}{3}$ is a solution of the quadratic equation $7x^2 + mx - 3 = 0$; find the value of m.

29. If $x = -3$ and $x = \frac{2}{3}$ are solutions of quadratic equation $mx^2 + 7x + n = 0$, find the values of m and n.

30. If quadratic equation $x^2 - (m + 1)x + 6 = 0$ has one root as $x = 3$; find the value of m and the other root of the equation.

31. Given that 2 is a root of the equation $3x^2 - p(x + 1) = 0$ and that the equation $px^2 - qx + 9 = 0$ has equal roots, find the values of p and q.

32. Solve : $\frac{x}{a} - \frac{a+b}{x} = \frac{b(a+b)}{ax}$.

33. Solve : $\left(\frac{1200}{x} + 2\right)(x - 10) - 1200 = 60$.

34. If -1 and 3 are the roots of $x^2 + px + q = 0$, find the values of p and q.

5.4 Solving quadratic equations using the formula :

The roots of the quadratic equation $ax^2 + bx + c = 0$; where $a \neq 0$ can be obtained by using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof :

Given : $ax^2 + bx + c = 0$

$\Rightarrow$ $4a^2x^2 + 4abx + 4ac = 0$ [On multiplying each term by $4a$]

$\Rightarrow$ $(2ax)^2 + 2 \times 2ax \times b + b^2 - b^2 + 4ac = 0$

$\Rightarrow$ $(2ax + b)^2 - b^2 + 4ac = 0$

$\Rightarrow$ $(2ax + b)^2 = b^2 - 4ac$

$$\Rightarrow \quad 2ax + b = \pm\sqrt{b^2 - 4ac}$$

$$\Rightarrow \quad 2ax = -b \pm\sqrt{b^2 - 4ac} \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ **Ans.**

13 Solve each of the following equations by using the formula :

(i) $5x^2 - 2x - 3 = 0$ (ii) $x^2 = 18x - 77$ (iii) $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0.$

Solution :

(i) Comparing $5x^2 - 2x - 3 = 0$ with $ax^2 + bx + c = 0$, we get :

$a = 5$, $b = -2$ and $c = -3$;

and so, $x = \dfrac{2 \pm \sqrt{(-2)^2 - 4\times5\times-3}}{2\times5}$ $\left[\because x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$

$= \dfrac{2\pm\sqrt{64}}{10} = \dfrac{2\pm8}{10} = \dfrac{2+8}{10}$, or $\dfrac{2-8}{10} = 1$, or $-\dfrac{3}{5}$ **Ans.**

(ii) $x^2 = 18x - 77 = 0 \Rightarrow x^2 - 18x + 77 = 0$

Comparing with $ax^2 + bx + c = 0$, we get : $a = 1$, $b = -18$ and $c = 77$

$\therefore x = \dfrac{18\pm\sqrt{(-18)^2 - 4\times1\times77}}{2\times1}$ $\left[\because x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$

$= \dfrac{18\pm\sqrt{16}}{2} = \dfrac{18+4}{2}$, or $\dfrac{18-4}{2} = 11$, or 7 **Ans.**

(iii) $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0 \Rightarrow a = \sqrt{3}$, $b = 11$ and $c = 6\sqrt{3}$

$\therefore x = \dfrac{-11\pm\sqrt{(11)^2 - 4\times\sqrt{3}\times6\sqrt{3}}}{2\times\sqrt{3}}$ $\left[\because x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$

$= \dfrac{-11\pm\sqrt{49}}{2\sqrt{3}} = \dfrac{-11+7}{2\sqrt{3}}$, or $\dfrac{-11-7}{2\sqrt{3}}$

$= \dfrac{-4}{2\sqrt{3}}$, or $\dfrac{-18}{2\sqrt{3}}$

$= -\dfrac{2}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$, or $\dfrac{-9}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$ [Rationalizing the denominators]

$= -\dfrac{2\sqrt{3}}{3}$, or $-3\sqrt{3}$ **Ans.**

14 Solve each of the following equations for x and give, in each case, your answer correct to 2 decimal places :

(i) $x^2 - 10x + 6 = 0$ (ii) $3x^2 + 5x - 9 = 0$

Solution :

(i) $x^2 - 10x + 6 = 0 \Rightarrow a = 1, b = -10$ and $c = 6$

$\therefore\ b^2 - 4ac = (-10)^2 - 4 \times 1 \times 6$

$= 100 - 24 = 76$

$\Rightarrow \sqrt{b^2-4ac} = \sqrt{76} = 8{\cdot}718$

$\therefore\ x = \dfrac{10 \pm 8{\cdot}718}{2\times1}$ $\quad [\because\ x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}]$

$= \dfrac{10+8{\cdot}718}{2}$, or $\dfrac{10-8{\cdot}718}{2}$

$= 9{\cdot}359$, or $0{\cdot}641$

$=$ **9·36**, or **0·64** [Correct to 2 decimal places] **Ans.**

(ii) $3x^2 + 5x - 9 = 0 \Rightarrow a = 3, b = 5$ and $c = -9$

$\therefore\ b^2 - 4ac = (5)^2 - 4 \times 3 \times -9$

$= 25 + 108 = 133$

$\Rightarrow \sqrt{b^2-4ac} = \sqrt{133} = 11{\cdot}533$

$\therefore\ x = \dfrac{-5\pm 11{\cdot}533}{2\times3}$ $\quad [\because\ x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}]$

$= \dfrac{-5+11{\cdot}533}{6}$, or $\dfrac{-5-11{\cdot}533}{6}$

$= 1{\cdot}089$, or $-2{\cdot}756$

$=$ **1·09**, or **–2·76** [Correct to 2 decimal places] **Ans.**

15 Solve the following equation :

$x - \dfrac{18}{x} = 6$. Give your answer correct to two significant figures. [2011]

Solution :

$x - \dfrac{18}{x} = 6 \Rightarrow x^2 - 18 = 6x$

$\Rightarrow x^2 - 6x - 18 = 0$

Comparing with $ax^2 + bx + c = 0$, we get : $a = 1$, $b = -6$ and $c = -18$.

$\therefore\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{6 \pm \sqrt{36 - 4 \times 1 \times -18}}{2 \times 1} = \frac{6 \pm 10 \cdot 392}{2}$$

$$= \frac{16 \cdot 392}{2} \text{ or } \frac{-4 \cdot 392}{2}$$

$= 8 \cdot 196$ or $-2 \cdot 196 =$ **$8 \cdot 2$ or $-2 \cdot 2$** **Ans.**

5.5 Equations Reducible to Quadratic Equations :

16 Solve : (i) $2x^4 - 5x^2 + 3 = 0$ (ii) $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0, x \in R$

Solution :

(i) $2x^4 - 5x^2 + 3 = 0$

$\Rightarrow 2y^2 - 5y + 3 = 0$ [Taking $x^2 = y$]

$\Rightarrow (y - 1)(2y - 3) = 0$ [On factorising]

$\Rightarrow y = 1$, or $y = \frac{3}{2}$

When $y = 1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

and, when $y = \frac{3}{2} \Rightarrow x^2 = \frac{3}{2} \Rightarrow x = \pm \sqrt{\frac{3}{2}} = \pm \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \pm \frac{\sqrt{6}}{2}$

$\therefore$ **Required solution = $1, -1, \frac{\sqrt{6}}{2}$, or $-\frac{\sqrt{6}}{2}$** **Ans.**

(ii) $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0$

$\Rightarrow y^2 - y - 6 = 0$ [Taking $x^2 + 3x = y$]

$\Rightarrow (y - 3)(y + 2) = 0$ [On factorising]

$\Rightarrow y = 3$, or $y = -2$

$y = 3 \Rightarrow x^2 + 3x = 3$

$\Rightarrow x^2 + 3x - 3 = 0$

$$\Rightarrow x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1 \times -3}}{2 \times 1} \Rightarrow x = \frac{-3 \pm \sqrt{21}}{2}$$

and $y = -2 \Rightarrow x^2 + 3x = -2 \Rightarrow x^2 + 3x + 2 = 0$

$$\Rightarrow x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$= \frac{-3 \pm 1}{2} = \frac{-3 + 1}{2} \text{ or } \frac{-3 - 1}{2} = -1 \text{ or } -2$$

$\therefore$ **Required solution is : $\frac{-3 + \sqrt{21}}{2}, \frac{-3 - \sqrt{21}}{2}, -1$, or -2** **Ans.**

17 Solve : $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$, $x \neq 0$ and $x \neq 1$.

Solution :

Let $\sqrt{\frac{x}{1-x}} = y \Rightarrow \sqrt{\frac{1-x}{x}} = \frac{1}{y}$

∴ Given equation reduces to :

$$y + \frac{1}{y} = \frac{13}{6} \Rightarrow 6y^2 + 6 = 13y$$
$$\Rightarrow 6y^2 - 13y + 6 = 0$$
$$\Rightarrow (2y - 3)(3y - 2) = 0 \qquad \text{[On factorising]}$$
$$\Rightarrow y = \frac{3}{2}, \text{ or } y = \frac{2}{3}$$

When $y = \frac{3}{2} \Rightarrow \sqrt{\frac{x}{1-x}} = \frac{3}{2} \Rightarrow \frac{x}{1-x} = \frac{9}{4}$

$$\Rightarrow 4x = 9 - 9x \Rightarrow x = \frac{9}{13}$$

and $y = \frac{2}{3} \Rightarrow \sqrt{\frac{x}{1-x}} = \frac{2}{3} \Rightarrow \frac{x}{1-x} = \frac{4}{9}$

$$\Rightarrow 9x = 4 - 4x \Rightarrow x = \frac{4}{13}$$

∴ **Required solution is : $\frac{9}{13}$, or $\frac{4}{13}$** **Ans.**

EXERCISE 5(C)

1. Solve, each of the following equations, using the formula :

(i) $x^2 - 6x = 27$ (ii) $x^2 - 10x + 21 = 0$

(iii) $x^2 + 6x - 10 = 0$ (iv) $x^2 + 2x - 6 = 0$

(v) $3x^2 + 2x - 1 = 0$ (vi) $2x^2 + 7x + 5 = 0$

(vii) $\frac{2}{3}x = -\frac{1}{6}x^2 - \frac{1}{3}$ (viii) $\frac{1}{15}x^2 + \frac{5}{3} = \frac{2}{3}x$

(ix) $x^2 - 6 = 2\sqrt{2}\,x$ (x) $\frac{4}{x} - 3 = \frac{5}{2x+3}$

(xi) $\frac{2x+3}{x+3} = \frac{x+4}{x+2}$

(xii) $\sqrt{6}\,x^2 - 4x - 2\sqrt{6} = 0$

(xiii) $\frac{2x}{x-4} + \frac{2x-5}{x-3} = 8\frac{1}{3}$

(xiv) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

2. Solve each of the following equations for x and give, in each case, your answer correct to one decimal place :

(i) $x^2 - 8x + 5 = 0$

(ii) $5x^2 + 10x - 3 = 0$

3. Solve each of the following equations for x and give, in each case, your answer correct to 2 decimal places :

(i) $2x^2 - 10x + 5 = 0$

(ii) $4x + \frac{6}{x} + 13 = 0$

(iii) $x^2 - 3x - 9 = 0$ **[2007]**

(iv) $x^2 - 5x - 10 = 0$ **[2013]**

4. Solve each of the following equations for x, giving your answer correct to 3 decimal places:

   (i) $3x^2 - 12x - 1 = 0$

   (ii) $x^2 - 16x + 6 = 0$

   (iii) $2x^2 + 11x + 4 = 0$

5. Solve :

   (i) $x^4 - 2x^2 - 3 = 0$ (ii) $x^4 - 10x^2 + 9 = 0$

6. Solve :

   (i) $(x^2 - x)^2 + 5(x^2 - x) + 4 = 0$

   (ii) $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$

7. Solve :

   (i) $\sqrt{\dfrac{x}{x-3}} + \sqrt{\dfrac{x-3}{x}} = \dfrac{5}{2}$

   (ii) $\left(\dfrac{2x-3}{x-1}\right) - 4\left(\dfrac{x-1}{2x-3}\right) = 3$

   (iii) $\left(\dfrac{3x+1}{x+1}\right) + \left(\dfrac{x+1}{3x+1}\right) = \dfrac{5}{2}$

8. Solve the equation $2x - \dfrac{1}{x} = 7$. Write your answer correct to two decimal places. **[2006]**

9. Solve the following equation and give your answer correct to 3 significant figures :

   $5x^2 - 3x - 4 = 0$ **[2012]**

10. Solve for x using the quadratic formula. Write your answer correct to two significant figures. $(x - 1)^2 - 3x + 4 = 0$. **[2014]**

18 Find the solution set of the equation $3x^2 - 8x - 3 = 0$; when :
(i) $x \in Z$ (integers) (ii) $x \in Q$ (rational numbers).

Solution :

$$3x^2 - 8x - 3 = 0 \Rightarrow 3x^2 - 9x + x - 3 = 0$$
$$\Rightarrow 3x(x-3) + 1(x-3) = 0$$
$$\Rightarrow (x-3)(3x+1) = 0$$
$$\Rightarrow x = 3, \text{ or } x = -\frac{1}{3}$$

(i) When $x \in Z$, the **solution set = {3}** **Ans.**

(ii) When $x \in Q$, the **solution set =** $\{3, -\frac{1}{3}\}$ **Ans.**

19 Solve : $(2x - 3)^2 = 25$.

Solution :

$$(2x-3)^2 = 25 \Rightarrow 4x^2 - 12x + 9 - 25 = 0$$
$$\Rightarrow 4x^2 - 12x - 16 = 0$$
$$\Rightarrow x^2 - 3x - 4 = 0$$
$$\Rightarrow (x-4)(x+1) = 0$$
$$\Rightarrow x = 4, \text{ or } x = -1$$ **Ans.**

Alternative method :

$$(2x-3)^2 = 25 \Rightarrow 2x - 3 = \pm 5$$
Now, $2x - 3 = 5 \Rightarrow 2x = 8$ and $x = 4$

And, $2x - 3 = -5 \Rightarrow 2x = -2$ and $x = -1$

$\therefore$ $x = 4$, or $x = -1$ **Ans.**

20 Solve for x : $4(x - \frac{1}{x})^2 + 8(x + \frac{1}{x}) = 29.\ x \neq 0.$

Solution :

$$\text{Let } x + \frac{1}{x} = y$$

$$\because\ (x + \frac{1}{x})^2 - (x - \frac{1}{x})^2 = 4 \quad \Rightarrow \quad y^2 - (x - \frac{1}{x})^2 = 4$$

$$\text{and } (x - \frac{1}{x})^2 = y^2 - 4$$

$$\therefore 4(x - \frac{1}{x})^2 + 8\,(x + \frac{1}{x}) = 29 \quad \Rightarrow \quad 4(y^2 - 4) + 8y = 29$$

$$\Rightarrow \quad 4y^2 - 16 + 8y = 29$$

$$\Rightarrow \quad 4y^2 + 8y - 45 = 0$$

$$\Rightarrow \quad 4y^2 + 18y - 10y - 45 = 0 \quad i.e.\ \ 2y(2y + 9) - 5(2y + 9) = 0$$

$$\Rightarrow \quad (2y + 9)\,(2y - 5) = 0 \qquad i.e.\ \ y = -\frac{9}{2} \text{ or } y = \frac{5}{2}$$

$$y = -\frac{9}{2} \quad \Rightarrow \quad x + \frac{1}{x} = -\frac{9}{2} \quad i.e.\ 2x^2 + 9x + 2 = 0$$

$$\Rightarrow \quad x = \frac{-9 \pm \sqrt{(9)^2 - 4\times2\times2}}{2\times2} = \frac{-9 \pm \sqrt{65}}{4}$$

$$y = \frac{5}{2} \quad \Rightarrow \quad x + \frac{1}{x} = \frac{5}{2} \quad i.e.\ \ 2x^2 - 5x + 2 = 0$$

$$\Rightarrow \quad 2x^2 - 4x - x + 2 = 0 \quad i.e.\ \ 2x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow \quad (x - 2)\,(2x - 1) = 0 \quad i.e.\ \ x = 2 \text{ or } x = \frac{1}{2}$$

$$\therefore \quad \textbf{Solution} = \frac{-9 \pm \sqrt{65}}{4},\ 2, \text{ or } \frac{1}{2}$$ **Ans.**

21 Solve : $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b$, where $a + b \neq 0$, $ab \neq 0$.

Solution :

$$\frac{a}{ax-1} + \frac{b}{bx-1} = a + b \Rightarrow \frac{a}{ax-1} - b + \frac{b}{bx-1} - a = 0$$

i.e.

$$\frac{a - abx + b}{ax-1} + \frac{b - abx + a}{bx-1} = 0$$

$$\Rightarrow \quad (a + b - abx)\left[\frac{1}{ax-1} + \frac{1}{bx-1}\right] = 0$$

$$\Rightarrow a + b - abx = 0, \qquad \text{or} \quad \frac{1}{ax-1} + \frac{1}{bx-1} = 0$$

$\Rightarrow \quad -abx = -a - b, \quad$ or $\quad \frac{1}{ax-1} = -\frac{1}{bx-1}$

$\Rightarrow \quad abx = a + b, \quad$ or $\quad bx - 1 = -ax + 1$

$\Rightarrow \quad x = \frac{a+b}{ab}, \quad$ or $\quad x = \frac{2}{a+b}$ **Ans.**

EXERCISE 5(D)

Solve each of the following equations :

1. $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0;$
   $x \neq 3, x \neq -\frac{3}{2}$
2. $(2x + 3)^2 = 81$
3. $a^2x^2 - b^2 = 0$
4. $x^2 - \frac{11}{4}x + \frac{15}{8} = 0$
5. $x + \frac{4}{x} = -4; x \neq 0$
6. $2x^4 - 5x^2 + 3 = 0$ **Take $x^2 = y$**
7. $x^4 - 2x^2 - 3 = 0$
8. $9(x^2 + \frac{1}{x^2}) - 9(x + \frac{1}{x}) - 52 = 0$

Let $x + \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} + 2 = y^2$

$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2.$

$\therefore$ Given equation reduces to :

$9(y^2 - 2) - 9y - 52 = 0$

i.e. $9y^2 - 9y - 70 = 0$

$\Rightarrow (3y - 10)(3y + 7) = 0$

$\Rightarrow y = \frac{10}{3}$, *or* $y = -\frac{7}{3}$.

$y = \frac{10}{3} \Rightarrow x + \frac{1}{x} = \frac{10}{3},$

solve it to get $x = 3$, or $\frac{1}{3}$.

Similarly $y = -\frac{7}{3} \Rightarrow x + \frac{1}{x} = -\frac{7}{3},$

solve it to get $x = \frac{-7 \pm \sqrt{13}}{6}$.

$\therefore$ The solution is $3, \frac{1}{3}, \frac{-7 \pm \sqrt{13}}{6}$

9. $2(x^2 + \frac{1}{x^2}) - (x + \frac{1}{x}) = 11$
10. $(x^2 + \frac{1}{x^2}) - 3(x - \frac{1}{x}) - 2 = 0$

Let $x - \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} = y^2 + 2$

11. $(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$

Take $x^2 + 5x = y$

12. Solve each of the following equations, giving answer upto two decimal places.
    (i) $x^2 - 5x - 10 = 0$ **[2005]**
    (ii) $3x^2 - x - 7 = 0$ **[2004]**
13. Solve: $\left(\frac{x}{x+2}\right)^2 - 7\left(\frac{x}{x+2}\right) + 12 = 0; x \neq -2.$
14. Solve :
    (i) $x^2 - 11x - 12 = 0$; when $x \in N$
    (ii) $x^2 - 4x - 12 = 0$; when $x \in I$
    (iii) $2x^2 - 9x + 10 = 0$; when $x \in Q$.
15. Solve :
    $(a + b)^2 x^2 - (a + b) x - 6 = 0; a + b \neq 0.$

Take : $(a + b) x = y$

16. Solve : $\frac{1}{p} + \frac{1}{q} + \frac{1}{x} = \frac{1}{x+p+q}$

Take : $\left(\frac{1}{p} + \frac{1}{q}\right) + \left(\frac{1}{x} - \frac{1}{x+p+q}\right) = 0$

17. Solve :
    (i) $x(x + 1) + (x + 2)(x + 3) = 42$
    (ii) $\frac{1}{x+1} - \frac{2}{x+2} = \frac{3}{x+3} - \frac{4}{x+4}$

18. For each equation, given below, find the value of 'm' so that the equation has equal roots. Also, find the solution of each equation :

(i) $(m - 3)x^2 - 4x + 1 = 0$

(ii) $3x^2 + 12x + (m + 7) = 0$

(iii) $x^2 - (m + 2)x + (m + 5) = 0$

19. Without solving the following quadratic equation, find the value of 'p' for which the roots are equal.

$px^2 - 4x + 3 = 0$ **[2010]**

20. Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.

$x^2 + 2(m - 1)x + (m + 5) = 0$ **[2012]**