

8

Remainder and Factor Theorems

8.1 A Basic Concept :

1. In equation $f(x) = 2x^2 - 5x - 7$, $f(x)$ is said to be a function of variable x as the value of $f(x)$ depends on the value of x . The following examples prove this statement.

$$f(x) = 2x^2 - 5x - 7 \Rightarrow$$

(i) if $x = 3$, $f(3) = 2 \times (3)^2 - 5 \times 3 - 7 = 18 - 22 = -4$

(ii) if $x = -3$, $f(-3) = 2(-3)^2 - 5(-3) - 7 = 18 + 15 - 7 = 26$ and so on.

In the same way :

in $f(y) = 2y^3 - 3y + 1$, $f(y)$ is a function of variable y as the value of $f(y)$ depends on the value of y , e.g.,

$$f(y) = 2y^3 - 3y + 1 \Rightarrow$$

(i) if $y = 5$, $f(5) = 2 \times 5^3 - 3 \times 5 + 1 = 250 - 15 + 1 = 236$.

(ii) if $y = -2$, $f(-2) = 2 \times (-2)^3 - 3 \times -2 + 1 = -16 + 6 + 1 = -9$ and so on.

2. Find the remainder obtained on dividing $f(x) = x^2 - 5x + 8$ by $x - 2$.

$$\begin{array}{r}
 x - 3 \\
 x - 2 \overline{) x^2 - 5x + 8} \\
 \underline{x^2 - 2x} \\
 - 3x + 8 \\
 \underline{- 3x + 6} \\
 + 2 \\
 \hline
 \text{The remainder} = \underline{2}
 \end{array}$$

Now, find $f(2)$

$$f(x) = x^2 - 5x + 8$$

$$\Rightarrow f(2) = (2)^2 - 5 \times 2 + 8$$

$$= 4 - 10 + 8 = 2$$

= **The remainder when $x^2 - 5x + 8$ is divided by $x - 2$.**

3. Again, find the remainder obtained on dividing $f(x) = 6x^3 - 3x^2 + 8x - 5$ by $x + 3$

$$\begin{array}{r}
 6x^2 - 21x + 71 \\
 x + 3 \overline{) 6x^3 - 3x^2 + 8x - 5} \\
 \underline{6x^3 + 18x^2} \\
 - 21x^2 + 8x \\
 \underline{- 21x^2 - 63x} \\
 + 71x - 5 \\
 71x + 213 \\
 \hline
 \text{The remainder} = \underline{-218}
 \end{array}$$

Now, find $f(-3)$

$$f(x) = 6x^3 - 3x^2 + 8x - 5$$

$$\Rightarrow f(-3) = 6(-3)^3 - 3(-3)^2 + 8(-3) - 5$$

$$= -162 - 27 - 24 - 5$$

$$= -218$$

= **The remainder when $6x^3 - 3x^2 + 8x - 5$ is divided by $x + 3$**

It is clear from the examples, given above, that :

1. when $f(x)$ is divided by $x - 2$, the remainder = the value of $f(2)$.
2. when $f(x)$ is divided by $x + 3$, the remainder = the value of $f(-3)$

The method of finding the remainder without actually performing the process of division is called **Remainder Theorem**.

8.2 Remainder Theorem :

If $f(x)$, a polynomial in x , is divided by $(x - a)$, the remainder = $f(a)$

e.g. If $f(x)$ is divided by $(x - 3)$, the remainder is $f(3)$.

For finding the remainder, using Remainder Theorem :

Step 1 : Put the divisor equal to zero and solve the equation obtained to get the value of its variable.

Step 2 : Substitute the value of the variable, obtained in step 1, in the given polynomial and simplify it to get the required remainder.

1 Find the remainder when $x^2 - 8x + 4$ is divided by $2x + 1$.

Solution :

Step 1 : $2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

Step 2 : Required remainder = Value of given polynomial $x^2 - 8x + 4$ at $x = -\frac{1}{2}$

$$\begin{aligned}\therefore \text{Remainder} &= \left(-\frac{1}{2}\right)^2 - 8\left(-\frac{1}{2}\right) + 4 \\ &= \frac{1}{4} + 4 + 4 = 8\frac{1}{4}\end{aligned}$$

Ans.

2 Find the value of 'a' if the division of $ax^3 + 9x^2 + 4x - 10$ by $x + 3$ leaves a remainder of 5.

Solution :

$$x + 3 = 0 \Rightarrow x = -3$$

Given, remainder is 5; therefore :

The value of $ax^3 + 9x^2 + 4x - 10$ at $x = -3$ is 5

$$\Rightarrow a(-3)^3 + 9(-3)^2 + 4(-3) - 10 = 5$$

$$\Rightarrow -27a + 81 - 12 - 10 = 5 \text{ or } a = 2$$

Ans.

3 When the polynomial $2x^3 - kx^2 + (5k - 3)x - 8$ is divided by $x - 2$, the remainder is 14. Find the value of 'k'.

Solution :

$$x - 2 = 0 \Rightarrow x = 2$$

Given, remainder is 14, therefore :

$$\Rightarrow 2(2)^3 - k(2)^2 + (5k - 3) \times 2 - 8 = 14$$

$$\Rightarrow 16 - 4k + 10k - 6 - 8 = 14$$

$$\Rightarrow 6k = 12 \text{ and } k = 2 \quad \text{Ans.}$$

- 4** The polynomials $3x^3 - ax^2 + 5x - 13$ and $(a + 1)x^2 - 7x + 5$ leave the same remainder when divided by $x - 3$. Find the value of 'a'.

Solution :

$$x - 3 = 0 \Rightarrow x = 3$$

Since, the given polynomials leave the same remainder when divided by $x - 3$

Value of polynomial $3x^3 - ax^2 + 5x - 13$ at $x = 3$ is the same as the value of polynomial $(a + 1)x^2 - 7x + 5$ at $x = 3$

$$\Rightarrow 3(3)^3 - a(3)^2 + 5 \times 3 - 13 = (a + 1)(3)^2 - 7 \times 3 + 5$$

$$\Rightarrow 81 - 9a + 15 - 13 = 9a + 9 - 21 + 5$$

$$\Rightarrow 18a = 90 \text{ and } a = 5 \quad \text{Ans.}$$

- 5** When $f(x) = x^3 + ax^2 - bx - 8$ is divided by $x - 2$, the remainder is zero and when divided by $x + 1$, the remainder is -30 . Find the values of 'a' and 'b'.

Solution :

$$\text{Since, } x - 2 = 0 \Rightarrow x = 2$$

And, given that on dividing $f(x) = x^3 + ax^2 - bx - 8$ by $x - 2$, the remainder = 0

$$\Rightarrow f(2) = 0$$

$$\Rightarrow (2)^3 + a(2)^2 - b(2) - 8 = 0 \quad \text{i.e. } 8 + 4a - 2b - 8 = 0$$

$$\Rightarrow 4a - 2b = 0 \quad \text{i.e. } 2a - b = 0 \quad \dots\text{I}$$

Again, given that on dividing $f(x) = x^3 + ax^2 - bx - 8$ by $x + 1$, the remainder = -30

$$\Rightarrow f(-1) = -30 \quad [x + 1 = 0 \Rightarrow x = -1]$$

$$\Rightarrow (-1)^3 + a(-1)^2 - b(-1) - 8 = -30 \quad \text{i.e. } -1 + a + b - 8 = -30$$

$$\Rightarrow a + b = -21 \quad \dots\text{II}$$

On solving equations I and II, we get : $a = -7$ and $b = -14$ **Ans.**

- 6** What number should be added to $2x^3 - 3x^2 + x$ so that when the resulting polynomial is divided by $x - 2$, the remainder is 3 ?

Solution :

Let the number added be k so the resulting polynomial is

$$2x^3 - 3x^2 + x + k$$

Given, when this polynomial is divided by $x - 2$, the remainder = 3

$$\Rightarrow 2(2)^3 - 3(2)^2 + 2 + k = 3 \quad [x - 2 = 0 \Rightarrow x = 2]$$

$$\Rightarrow 16 - 12 + 2 + k = 3 \text{ i.e., } k = -3$$

\therefore The required number to be added = -3

Ans.

8.3 Factor Theorem :

When a polynomial $f(x)$ is divided by $x - a$, the remainder = $f(a)$. And, if remainder $f(a) = 0$; $x - a$ is a factor of the polynomial $f(x)$.

For example :

Let $f(x) = x^2 - 5x + 6$ be divided by $x - 3$;

then remainder = $f(3)$

$$= (3)^2 - 5 \times 3 + 6 = 0$$

\therefore Remainder = 0

$\Rightarrow x - 3$ is a factor of $f(x) = x^2 - 5x + 6$

7 Determine whether $x - 1$ is a factor of $x^6 - x^5 + x^4 + x^3 - x^2 - x + 1$ or not ?

Solution :

$$x - 1 = 0 \Rightarrow x = 1$$

\therefore When given polynomial is divided by $x - 1$, the remainder

$$= (1)^6 - (1)^5 + (1)^4 + (1)^3 - (1)^2 - (1) + 1$$

$$= 1 - 1 + 1 + 1 - 1 - 1 + 1$$

$$= 4 - 3 = 1, \text{ which is not equal to zero.}$$

$\therefore x - 1$ is **not a factor** of the given polynomial.

Ans.

8 If $x - 2$ is a factor of $x^2 - 7x + 2a$, find the value of a .

Solution :

$$x - 2 = 0 \Rightarrow x = 2$$

Since, $x - 2$ is a factor of polynomial $x^2 - 7x + 2a$

$$\Rightarrow \text{Remainder} = 0 \Rightarrow (2)^2 - 7(2) + 2a = 0 \Rightarrow a = 5$$

Ans.

9 Find the value of 'k' if $(x - 2)$ is a factor of $x^3 + 2x^2 - kx + 10$.

Hence, determine whether $(x + 5)$ is also a factor.

[2011]

Solution :

$x - 2$ is a factor and $x - 2 = 0 \Rightarrow x = 2$

\therefore The value of given expression $x^3 + 2x^2 - kx + 10$ is zero at $x = 2$

i.e. remainder = 0

$$\Rightarrow (2)^3 + 2(2)^2 - k \times 2 + 10 = 0$$

$$\Rightarrow 8 + 8 - 2k + 10 = 0$$

$$\Rightarrow k = 13$$

Ans.

On substituting $k = 13$, the given expression becomes $x^3 + 2x^2 - 13x + 10$.

Now to check whether $(x + 5)$ is also a factor or not,

find the value of the given expression for $x = -5$. [$\because x + 5 = 0 \Rightarrow x = -5$]

$$\begin{aligned}\therefore x^3 + 2x^2 - 13x + 10 & \text{ (at } x = -5\text{)} \\ & = (-5)^3 + 2(-5)^2 - 13(-5) + 10 \\ & = -125 + 50 + 65 + 10 = -125 + 125 = 0\end{aligned}$$

Since, the remainder is 0 $\Rightarrow (x + 5)$ is a factor Ans.

- 10** Given that $x + 2$ and $x - 3$ are factors of $x^3 + ax + b$; calculate the values of a and b .

Solution :

Given, $x + 2$ is a factor of $x^3 + ax + b$;

$$\Rightarrow (-2)^3 + a(-2) + b = 0 \quad [x + 2 = 0 \Rightarrow x = -2]$$

$$\Rightarrow -2a + b = 8 \quad \dots\text{I}$$

Again, given that :

$x - 3$ is a factor of $x^3 + ax + b$;

$$\Rightarrow (3)^3 + a(3) + b = 0 \quad [x - 3 = 0 \Rightarrow x = 3]$$

$$\Rightarrow 3a + b = -27 \quad \dots\text{II}$$

On solving equations I and II, we get $a = -7$ and $b = -6$ Ans.

- 11** Polynomial $x^3 - ax^2 + bx - 6$ leaves remainder -8 when divided by $x - 1$ and $x - 2$ is a factor of it. Find the values of ' a ' and ' b '.

Solution :

On dividing by $x - 1$, the polynomial $x^3 - ax^2 + bx - 6$ leaves remainder -8

$$\Rightarrow (1)^3 - a(1)^2 + b(1) - 6 = -8 \quad [x - 1 = 0 \Rightarrow x = 1]$$

$$\Rightarrow -a + b = -3$$

$$\text{i.e.} \quad a - b = 3 \quad \dots\text{I}$$

$(x - 2)$ is a factor of polynomial $x^3 - ax^2 + bx - 6$

$$\Rightarrow (2)^3 - a(2)^2 + b(2) - 6 = 0 \quad [x - 2 = 0 \Rightarrow x = 2]$$

$$\Rightarrow 8 - 4a + 2b - 6 = 0$$

$$\text{i.e.} \quad 2a - b = 1 \quad \dots\text{II}$$

On solving equations I and II, we get :

$$a = -2 \quad \text{and} \quad b = -5 \quad \text{Ans.}$$

EXERCISE 8(A)

1. Find, in each case, the remainder when :
 - (i) $x^4 - 3x^2 + 2x + 1$ is divided by $x - 1$.
 - (ii) $x^3 + 3x^2 - 12x + 4$ is divided by $x - 2$.
 - (iii) $x^4 + 1$ is divided by $x + 1$.
2. Show that :
 - (i) $x - 2$ is a factor of $5x^2 + 15x - 50$.
 - (ii) $3x + 2$ is a factor of $3x^2 - x - 2$.
3. Use the Remainder Theorem to find which of the following is a factor of $2x^3 + 3x^2 - 5x - 6$.
 - (i) $x + 1$ (ii) $2x - 1$
 - (iii) $x + 2$
4. (i) If $2x + 1$ is a factor of $2x^2 + ax - 3$, find the value of a .
 (ii) Find the value of k , if $3x - 4$ is a factor of expression $3x^2 + 2x - k$.
5. Find the values of constants a and b when $x - 2$ and $x + 3$ both are the factors of expression $x^3 + ax^2 + bx - 12$.
6. Find the value of k , if $2x + 1$ is a factor of $(3k + 2)x^3 + (k - 1)$.
7. Find the value of a , if $x - 2$ is a factor of $2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$.
8. Find the values of m and n so that $x - 1$ and $x + 2$ both are factors of $x^3 + (3m + 1)x^2 + nx - 18$.
9. When $x^3 + 2x^2 - kx + 4$ is divided by $x - 2$, the remainder is k . Find the value of constant k .
10. Find the value of a , if the division of $ax^3 + 9x^2 + 4x - 10$ by $x + 3$ leaves a remainder 5.
11. If $x^3 + ax^2 + bx + 6$ has $x - 2$ as a factor and leaves a remainder 3 when divided by $x - 3$, find the values of a and b . [2005]
12. The expression $2x^3 + ax^2 + bx - 2$ leaves remainder 7 and 0 when divided by $2x - 3$ and $x + 2$ respectively. Calculate the values of a and b .
13. What number should be added to $3x^3 - 5x^2 + 6x$ so that when resulting polynomial is divided by $x - 3$, the remainder is 8 ?
14. What number should be subtracted from $x^3 + 3x^2 - 8x + 14$ so that on dividing it by $x - 2$, the remainder is 10 ?
15. The polynomials $2x^3 - 7x^2 + ax - 6$ and $x^3 - 8x^2 + (2a + 1)x - 16$ leave the same remainder when divided by $x - 2$. Find the value of 'a'.
16. If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b . [2013]
17. Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leave the same remainder when divided by $x + 3$. [2015]

8.4 Using the Factor Theorem to factorise the given polynomial :

[Factorising a polynomial completely after obtaining one factor by factor theorem]

When expression $f(x)$ is divided by $x - a$, the remainder = $f(a)$.

If the remainder $f(a) = 0$.

$$\Rightarrow x - a \text{ is a factor of expression } f(x).$$

Conversely, if for the expression $f(x)$, $f(a) = 0$; $\Rightarrow (x - a)$ is a factor.

For example :

(i) Let $f(x) = x^2 - 7x + 10$; then

$$f(2) = (2)^2 - 7 \times 2 + 10 = 0$$

$$\Rightarrow x - 2 \text{ is a factor of } f(x) = x^2 - 7x + 10$$

(ii) Let $f(x) = 2x^2 - x - 3$; then $f(-1) = 2(-1)^2 - (-1) - 3 = 0$

$$\Rightarrow x + 1 \text{ is a factor of } f(x) = 2x^2 - x - 3 \quad \text{and so on.}$$

- 12** Using the Factor Theorem, show that $(x - 2)$ is a factor of $3x^2 - 5x - 2$. Hence, factorise the given expression.

Solution :

$$\because x - 2 = 0 \Rightarrow x = 2$$

$$\begin{aligned} \therefore \text{Remainder} &= \text{The value of } 3x^2 - 5x - 2 \text{ at } x = 2 \\ &= 3(2)^2 - 5(2) - 2 \\ &= 12 - 10 - 2 = 0 \end{aligned}$$

$$\Rightarrow (x - 2) \text{ is a factor of } 3x^2 - 5x - 2$$

Now, dividing $(3x^2 - 5x - 2)$ by $(x - 2)$, we get quotient = $3x + 1$

$$\therefore 3x^2 - 5x - 2 = (x - 2)(3x + 1)$$

$$x - 2 \overline{) \begin{array}{r} 3x + 1 \\ 3x^2 - 5x - 2 \\ \underline{- \quad +} \\ x - 2 \\ x - 2 \\ \underline{- \quad +} \\ - 2 \\ - 2 \\ \underline{- \quad +} \\ 0 \end{array}}$$

Ans.

- 13** Show that $2x + 7$ is a factor of $2x^3 + 5x^2 - 11x - 14$. Hence, factorise the given expression completely, using the factor theorem. [2006]

Solution :

$$2x + 7 = 0 \Rightarrow x = -\frac{7}{2}$$

$$\text{Remainder} = \text{Value of } 2x^3 + 5x^2 - 11x - 14 \text{ at } x = -\frac{7}{2}$$

$$\begin{aligned} &= 2\left(-\frac{7}{2}\right)^3 + 5\left(-\frac{7}{2}\right)^2 - 11\left(-\frac{7}{2}\right) - 14 \\ &= -\frac{343}{4} + \frac{245}{4} + \frac{77}{2} - 14 \\ &= \frac{-343 + 245 + 154 - 56}{4} = 0 \end{aligned}$$

$$\Rightarrow (2x + 7) \text{ is a factor of } 2x^3 + 5x^2 - 11x - 14.$$

$$\therefore 2x^3 + 5x^2 - 11x - 14 = (2x + 7)(x^2 - x - 2)$$

$$= (2x + 7)(x^2 - 2x + x - 2)$$

$$= (2x + 7)[x(x - 2) + 1(x - 2)]$$

$$= (2x + 7)(x - 2)(x + 1)$$

$$2x + 7 \overline{) \begin{array}{r} x^2 - x - 2 \\ 2x^3 + 5x^2 - 11x - 14 \\ \underline{- \quad -} \\ - 2x^2 - 11x - 14 \\ - 2x^2 - 7x \\ - 4x - 14 \\ - 4x - 14 \\ \underline{+ \quad +} \\ - 14 - 14 \\ - 28 \\ - 28 \\ \underline{+ \quad +} \\ 0 \end{array}}$$

Ans.

- 14** Using the Remainder Theorem, factorise the expression $2x^3 + x^2 - 2x - 1$ completely.

Solution :

First Step : For $x = 1$, the value of given expression

$$= 2(1)^3 + (1)^2 - 2(1) - 1.$$

$$= 2 + 1 - 2 - 1 = 0$$

$$\Rightarrow x - 1 \text{ is a factor of } 2x^3 + x^2 - 2x - 1$$

$$x - 1 \overline{) \begin{array}{r} 2x^2 + 3x + 1 \\ 2x^3 + x^2 - 2x - 1 \\ \underline{- \quad +} \\ 3x^2 - 2x - 1 \\ 3x^2 - 3x \\ - 2x - 1 \\ - 2x - 1 \\ \underline{- \quad +} \\ - 1 - 1 \\ - 2 \\ - 2 \\ \underline{- \quad +} \\ 0 \end{array}}$$

Second Step :

$$\begin{aligned}2x^3 + x^2 - 2x - 1 &= (x - 1)(2x^2 + 3x + 1) \\ &= (x - 1)(2x^2 + 2x + x + 1) \\ &= (x - 1)[2x(x + 1) + 1(x + 1)] \\ &= (x - 1)(x + 1)(2x + 1)\end{aligned}$$

Ans.

- 15** Find the values of 'a' and 'b' so that the polynomial $x^3 + ax^2 + bx - 45$ has $(x - 1)$ and $(x + 5)$ as its factors.
For the values of 'a' and 'b', as obtained above, factorise the given polynomial completely.

Solution :

$(x - 1)$ is a factor of given polynomial $x^3 + ax^2 + bx - 45$

$$\Rightarrow (1)^3 + a(1)^2 + b(1) - 45 = 0 \quad [x - 1 = 0 \Rightarrow x = 1]$$

$$\text{i.e.} \quad a + b = 44 \quad \dots\text{I}$$

$(x + 5)$ is a factor of given polynomial

$$\Rightarrow (-5)^3 + a(-5)^2 + b(-5) - 45 = 0 \quad [x + 5 = 0 \Rightarrow x = -5]$$

$$\Rightarrow -125 + 25a - 5b - 45 = 0$$

$$\text{i.e.} \quad 5a - b = 34 \quad \dots\text{II}$$

On solving equations I and II, we get :

$$a = 13 \quad \text{and} \quad b = 31 \quad \text{Ans.}$$

\therefore The given polynomial $x^3 + ax^2 + bx - 45$

$$= x^3 + 13x^2 + 31x - 45$$

Now divide this polynomial

by $(x - 1)$ as shown alongside :

$$\therefore x^3 + 13x^2 + 31x - 45$$

$$= (x - 1)(x^2 + 14x + 45)$$

$$= (x - 1)(x^2 + 9x + 5x + 45)$$

$$= (x - 1)[x(x + 9) + 5(x + 9)]$$

$$= (x - 1)(x + 9)(x + 5)$$

$$\begin{array}{r} x^2 + 14x + 45 \\ x - 1 \overline{) x^3 + 13x^2 + 31x - 45} \\ \underline{- +} \\ 14x^2 + 31x - 45 \\ \underline{14x^2 - 14x} \\ 45x - 45 \\ \underline{45x - 45} \\ 0 \end{array}$$

Ans.

- 16** If $(x - 2)$ is a factor of $2x^3 - x^2 - px - 2$
(i) find the value of p .
(ii) with the value of p , factorise the above expression completely. [2008]

Solution :

$$(i) \quad x - 2 = 0 \Rightarrow x = 2$$

Since, $(x - 2)$ is a factor of given expression

$$\therefore \text{Remainder} = 0$$

$$\Rightarrow 2(2)^3 - (2)^2 - p \times 2 - 2 = 0$$

$$\Rightarrow 10 - 2p = 0 \quad \text{and} \quad p = 5$$

Ans.

$$(ii) \therefore 2x^3 - x^2 - px - 2 = 2x^3 - x^2 - 5x - 2$$

On dividing $2x^3 - x^2 - 5x - 2$

by $x - 2$, we get :

$$\text{quotient} = 2x^2 + 3x + 1$$

$$\therefore 2x^3 - x^2 - 5x - 2$$

$$= (x - 2)(2x^2 + 3x + 1)$$

$$= (x - 2)(2x^2 + 2x + x + 1)$$

$$= (x - 2)[2x(x + 1) + 1(x + 1)]$$

$$= (x - 2)(x + 1)(2x + 1)$$

$$\begin{array}{r}
 2x^2 + 3x + 1 \\
 x - 2 \overline{) 2x^3 - x^2 - 5x - 2} \\
 \underline{2x^3 - 4x^2} \\
 + 3x^2 - 5x - 2 \\
 \underline{- 3x^2 + 6x} \\
 - x - 2 \\
 \underline{- x + 2} \\
 0
 \end{array}$$

Ans.

EXERCISE 8(B)

1. Using the Factor Theorem, show that :

(i) $(x - 2)$ is a factor of $x^3 - 2x^2 - 9x + 18$.

Hence, factorise the expression $x^3 - 2x^2 - 9x + 18$ completely.

(ii) $(x + 5)$ is a factor of $2x^3 + 5x^2 - 28x - 15$.

Hence, factorise the expression $2x^3 + 5x^2 - 28x - 15$ completely.

(iii) $(3x + 2)$ is a factor of $3x^3 + 2x^2 - 3x - 2$.

Hence, factorise the expression $3x^3 + 2x^2 - 3x - 2$ completely.

2. Using the Remainder Theorem, factorise each of the following completely :

(i) $3x^3 + 2x^2 - 19x + 6$ [2012]

(ii) $2x^3 + x^2 - 13x + 6$

(iii) $3x^3 + 2x^2 - 23x - 30$

(iv) $4x^3 + 7x^2 - 36x - 63$

(v) $x^3 + x^2 - 4x - 4$. [2004]

3. Using the Remainder Theorem, factorise the expression $3x^3 + 10x^2 + x - 6$. Hence, solve the equation $3x^3 + 10x^2 + x - 6 = 0$

4. Factorise the expression

$$f(x) = 2x^3 - 7x^2 - 3x + 18.$$

Hence, find all possible values of x for which $f(x) = 0$.

5. Given that $x - 2$ and $x + 1$ are factors of $f(x) = x^3 + 3x^2 + ax + b$; calculate the values of a and b . Hence, find all the factors of $f(x)$.

6. The expression $4x^3 - bx^2 + x - c$ leaves remainders 0 and 30 when divided by $x + 1$ and $2x - 3$ respectively. Calculate the values of b and c . Hence, factorise the expression completely.

7. If $x + a$ is a common factor of expressions $f(x) = x^2 + px + q$ and $g(x) = x^2 + mx + n$; show that : $a = \frac{n - q}{m - p}$

8. The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$, when divided by $x - 4$, leave the same remainder in each case. Find the value of a .

9. Find the value of 'a', if $(x - a)$ is a factor of $x^3 - ax^2 + x + 2$. [2003]

10. Find the number that must be subtracted from the polynomial $3y^3 + y^2 - 22y + 15$, so that the resulting polynomial is completely divisible by $y + 3$.

EXERCISE 8(C)

1. Show that $(x - 1)$ is a factor of $x^3 - 7x^2 + 14x - 8$.

Hence, completely factorise the given expression.

2. Using Remainder Theorem, factorise :
 $x^3 + 10x^2 - 37x + 26$ completely. [2014]
3. When $x^3 + 3x^2 - mx + 4$ is divided by $x - 2$, the remainder is $m + 3$. Find the value of m .
4. What should be subtracted from $3x^3 - 8x^2 + 4x - 3$, so that the resulting expression has $x + 2$ as a factor ?

The number to be subtracted
= Remainder obtained on dividing
 $3x^3 - 8x^2 + 4x - 3$ by $x + 2$.

5. If $(x + 1)$ and $(x - 2)$ are factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, find the values of a and b . And then, factorise the given expression completely.
6. If $x - 2$ is a factor of $x^2 + ax + b$ and $a + b = 1$, find the values of a and b .
7. Factorise $x^3 + 6x^2 + 11x + 6$ completely using factor theorem.
8. Find the value of ' m ', if $mx^3 + 2x^2 - 3$ and $x^2 - mx + 4$ leave the same remainder when each is divided by $x - 2$.
9. The polynomial $px^3 + 4x^2 - 3x + q$ is completely divisible by $x^2 - 1$; find the values of p and q . Also, for these values of p and q factorize the given polynomial completely.
10. Find the number which should be added to $x^2 + x + 3$ so that the resulting polynomial is completely divisible by $(x + 3)$.
11. When the polynomial $x^3 + 2x^2 - 5ax - 7$ is divided by $(x - 1)$, the remainder is A and when the polynomial $x^3 + ax^2 - 12x + 16$ is divided by $(x + 2)$, the remainder is B . Find the value of ' a ' if $2A + B = 0$.
12. $(3x + 5)$ is a factor of the polynomial $(a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15$. Find the value of ' a '. For this value of ' a ', factorise the given polynomial completely.
13. When divided by $x - 3$ the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of ' p '. [2010]
14. Use the Remainder Theorem to factorise the following expression :
 $2x^3 + x^2 - 13x + 6$ [2010]