11 Geometric Progression

11.1 Introduction :

A sequence is an arrangement of numbers written in a definite order according to a certain given rule.

(i) 2, 6, 18, 54, ... is a sequence in which each term multiplied by 3 gives its next term.

(ii) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}$, ... is a sequence in which each term multiplied by $-\frac{1}{2}$ gives its next term.

(iii) 6, 4, 2, 0, ... is a sequence in which each term decreased by 2 gives its next term.

When the members (terms) of a sequence are connected using a positive/negative sign, we get a series.

Each of following is a series :

(i) $2 + 6 + 18 + 54 +$ .... (ii) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} +$ ....

(iii) $6 + 4 + 2 + 0 +$ .... , etc.

In general, a sequence and a series are considered to be the same.

11.2 Geometric progression :

A sequence, in which each of its terms can be obtained by multiplying or dividing its preceding term by a fixed quantity, is called a **geometric progression** and **the fixed quantity** is called **common ratio**.

In general, **geometric** (geometrical) **progression** is abbreviated as **G.P.** and **common ratio** is denoted by letter ***r***.

(i) Sequence 2, 4, 8, 16, ... is a G.P. as each term of it multiplied by 2 gives its next term. [Clearly, common ratio $r = 2$].

(ii) Sequence $6, 2, \frac{2}{3}$, ... is a G.P. as each of its terms can be obtained by multiplying its preceding term by $\frac{1}{3}$. [Clearly, common ratio $r = \frac{1}{3}$].

(iii) $60, -24, 9\frac{3}{5}, -3\frac{21}{25}$, ... is a G.P. with common ratio $-\frac{2}{5}$.

In a G.P., common ratio is obtained by dividing any term of it by its preceding term.

For G.P. 16, 8, 4, 2, 1, $\frac{1}{2}$, ...

$$\frac{8}{16} = \frac{1}{2}, \quad \frac{4}{8} = \frac{1}{2}, \quad \frac{2}{4} = \frac{1}{2}, \ldots\ldots$$

$$\therefore \quad \frac{8}{16} = \frac{4}{8} = \frac{2}{4} = \ldots\ldots = \frac{1}{2} = \text{common ratio } (r)$$

If first term of a G.P. is 5 and its common ratio is 3, then

G.P. = 5, 5 × 3, 5 × 3 × 3, 5 × 3 × 3 × 3, ...

= 5, 15, 45, 135, ...

Which can also be written as : 5 + 15 + 45 + 135 + ...

11.3 General term of a geometric progression :

Let the first term of a geometric progression = a and its common ratio = r, then its :

first term, $t_1 = a = ar^{1-1}$,

second term, $t_2 = ar = ar^{2-1}$,

third term, $t_3 = ar^2 = ar^{3-1}$,

fourth term, $t_4 = ar^3 = ar^{4-1}$

....

....

....

and n^{th} term, $t_n = ar^{n-1}$

> First term = a,
> common ratio = r and
> n^{th} term = ar^{n-1}

If a geometric progression has n terms only, its n^{th} term is called its **last term** which is denoted by letter ***l***.

∴ Last term (l) = ar^{n-1}

i.e. n^{th} term = ar^{n-1}

⇒ (i) 8^{th} term = $ar^{8-1} = ar^7$,

(ii) 15^{th} term = ar^{14},

(iii) 20^{th} term = ar^{19} and so on.

> If out of the four quantities a, r, n and t_n (*i.e.* l) any three quantities are given, we can find the remaining fourth quantity.

1 Find which of the following is a G.P. :

(i) 2, $2\sqrt{2}$, 4, $4\sqrt{2}$, ...   (ii) $\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}$, ...

(iii) 4, 8, 16, ...   (iv) xy, x^2y, x^3y, ...

Solution :

(i) Since, $\frac{2\sqrt{2}}{2} = \sqrt{2}, \ \frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}, \ \frac{4\sqrt{2}}{4} = \sqrt{2}$

$$\Rightarrow \frac{2\sqrt{2}}{2} = \frac{4}{2\sqrt{2}} = \frac{4\sqrt{2}}{4} = \ldots\ldots\ldots\ldots = \sqrt{2}$$

$\therefore$ 2, $2\sqrt{2}$, 4, $4\sqrt{2}$, ... is a G.P. with common ratio $\sqrt{2}$. **Ans.**

(ii) Since, $\dfrac{\frac{2}{3}}{\frac{1}{3}} \neq \dfrac{\frac{1}{2}}{\frac{2}{3}}$, given sequence is not a G.P.

(iii) Since, $\dfrac{8}{4} = \dfrac{16}{8} = \ldots\ldots\ldots\ldots = 2$,

$\therefore$ Sequence 4, 8, 16, ... forms a G.P. with common ratio 2. **Ans.**

(iv) It is a G.P. as

$$\frac{x^2y}{xy} = \frac{x^3y}{x^2y} = \ldots\ldots\ldots\ldots = x.$$ **Ans.**

2 Find the 8[th] term of the geometric progression : 5, 10, 20, ...

Solution :

Clearly, first term $(a) = 5$

and, common ratio $(r) = \dfrac{10}{5} = 2$

$\therefore$ $t_n = ar^{n-1} \Rightarrow t_8 = 5 \times 2^{8-1}$

$= 5 \times 2^7 = 5 \times 128 = 640$ **Ans.**

3 Find the 19[th] term of the series : $\sqrt{3} + \dfrac{1}{\sqrt{3}} + \dfrac{1}{3\sqrt{3}} + \ldots\ldots\ldots\ldots$.

Solution :

In the given series

$$\frac{t_2}{t_1} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3},\ \frac{t_3}{t_2} = \frac{\frac{1}{3\sqrt{3}}}{\frac{1}{\sqrt{3}}} = \frac{1}{3} \quad \text{and so on.}$$

$\Rightarrow \sqrt{3} + \dfrac{1}{\sqrt{3}} + \dfrac{1}{3\sqrt{3}} + \ldots\ldots\ldots\ldots$ is a G.P. with common ratio $\dfrac{1}{3}$.

Since, first term $a = \sqrt{3}$, common ratio $r = \dfrac{1}{3}$ and $n = 19$

$\therefore$ **19[th] term** $= ar^{18}$ [$\because$ n^{th} term $= ar^{n-1}$]

$$= \sqrt{3} \times \left(\frac{1}{3}\right)^{18} = \frac{\sqrt{3}}{3^{18}}$$ **Ans.**

4 If the first two consecutive terms of a G.P. are 125 and 25, find its 6[th] term.

Solution :

Clearly, first term $a = 125$ and second term $ar = 25$

$\therefore$ Common ratio $= \frac{ar}{a} = \frac{25}{125} \Rightarrow r = \frac{1}{5}$

$\therefore$ **6th term of the G.P.** $= ar^5$

$$= 125 \times \left(\frac{1}{5}\right)^5 = \frac{125}{3125} = \mathbf{\frac{1}{25}}$$ **Ans.**

5 Find the next three terms of the sequence : 36, 12, 4, ...

Solution :

Since, $\frac{12}{36} = \frac{1}{3}, \frac{4}{12} = \frac{1}{3}, \ldots\ldots\ldots\ldots$

i.e., $\frac{12}{36} = \frac{4}{12} \Rightarrow 36, 12, 4, \ldots\ldots\ldots\ldots$ form a G.P.

Clearly, first term $a = 36$ and common ratio $r = \frac{1}{3}$

$\therefore$ **Next three terms** = 4th term, 5th term and 6th term

$= ar^3, ar^4$ and ar^5

$= 36 \times \left(\frac{1}{3}\right)^3, 36 \times \left(\frac{1}{3}\right)^4$ and $36 \times \left(\frac{1}{3}\right)^5$

$= \mathbf{\frac{4}{3}, \frac{4}{9}}$ **and** $\mathbf{\frac{4}{27}}$ **Ans.**

EXERCISE 11(A)

1. Find, which of the following sequences form a G.P. :
   (i) 8, 24, 72, 216, ...
   (ii) $\frac{1}{8}, \frac{1}{24}, \frac{1}{72}, \frac{1}{216}, \ldots\ldots\ldots\ldots$
   (iii) 9, 12, 16, 24, ...
2. Find the 9th term of the series :
   1, 4, 16, 64, ...
3. Find the seventh term of the G.P. :
   $1, \sqrt{3}, 3, 3\sqrt{3}, \ldots\ldots\ldots\ldots$.
4. Find the 8th term of the sequence :
   $\frac{3}{4}, 1\frac{1}{2}, 3, \ldots\ldots\ldots\ldots$.
5. Find the 10th term of the G.P. :
   $12, 4, 1\frac{1}{3}, \ldots\ldots\ldots\ldots$.
6. Find the n^{th} term of the series :
   1, 2, 4, 8, ...
7. Find the next three terms of the sequence :
   $\sqrt{5}, 5, 5\sqrt{5}, \ldots\ldots\ldots\ldots$.
8. Find the sixth term of the series :
   $2^2, 2^3, 2^4, \ldots\ldots\ldots\ldots$.
9. Find the seventh term of the G.P. :
   $\sqrt{3} + 1, 1, \frac{\sqrt{3}-1}{2}, \ldots\ldots\ldots\ldots$.
10. Find the G.P. whose first term is 64 and next term is 32.
11. Find the next three terms of the series :
    $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots\ldots\ldots\ldots$.
12. Find the next two terms of the series :
    $2 - 6 + 18 - 54$ ....

6 Find which term of G.P. 3 - 6 + 12 - 24 + ... is -384 ?

Solution :

For the given G.P. : first term $a = 3$, common ratio $r = -\frac{6}{3} = -2$.

If -384 is the n^{th} term of the given G.P.,

then $-384 = ar^{n-1} \Rightarrow -384 = 3 \times (-2)^{n-1}$

$$\Rightarrow -\frac{384}{3} = (-2)^{n-1} \text{ i.e. } (-2)^{n-1} = -128$$

$$\Rightarrow (-2)^{n-1} = -2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = (-2)^7$$

$$\Rightarrow n - 1 = 7 \text{ and } n = 8$$

$\therefore$ **-384 is 8^{th} term of the given G.P.** **Ans.**

7 Find the G.P. whose 5^{th} term is 48 and 8^{th} term is 384.

Solution :

Let the first term of the G.P. be a and its common ratio be r.

5^{th} term $= 48 \Rightarrow ar^4 = 48$

and, 8^{th} term $= 384 \Rightarrow ar^7 = 384$

$$\therefore \frac{ar^7}{ar^4} = \frac{384}{48} \Rightarrow r^3 = 8 \text{ i.e. } r = 2$$

$$ar^4 = 48 \Rightarrow a(2)^4 = 48$$

$$\Rightarrow a = \frac{48}{16} = 3$$

$\therefore$ **G.P.** $= a, ar, ar^2, \ldots\ldots\ldots\ldots$

$= 3, 3 \times 2, 3 \times 2^2, \ldots\ldots\ldots\ldots$

$=$ **3, 6, 12, ...** **Ans.**

8 If the 3^{rd} term of a G.P. is 4, find the product of its first five terms.

Solution :

Let first term of the G.P. be a and its common ratio be r.

Given 3^{rd} term of the G.P. $= 4 \Rightarrow ar^2 = 4$

Product of first five terms of this G.P.

$= a \times ar \times ar^2 \times ar^3 \times ar^4$

$= a^5r^{10} = (ar^2)^5 = (4)^5 =$ **1024** **Ans.**

9 The first term of a G.P. is 1. The sum of its third and fifth terms is 90. Find the common ratio of the G.P.

Solution :

Given first term of the G.P. = 1 *i.e.* $a = 1$

Let the common ratio be r

Since, 3rd term + 5th term = 90

$\Rightarrow \quad ar^2 + ar^4 = 90 \quad$ *i.e.* $\quad r^2 + r^4 = 90 \qquad [\because a = 1]$

$\Rightarrow \quad r^4 + r^2 - 90 = 0 \quad$ *i.e.* $\quad r^4 + 10r^2 - 9r^2 - 90 = 0$

$\Rightarrow \quad r^2(r^2 + 10) - 9(r^2 + 10) = 0$

$\Rightarrow \quad (r^2 + 10)(r^2 - 9) = 0 \quad$ *i.e.* $\quad r^2 + 10 = 0$ or $r^2 - 9 = 0$

$\Rightarrow \quad r^2 = 9 \qquad [\because r^2 \neq -10]$

$\Rightarrow \quad r = \pm 3$

$\therefore$ **Common ratio of the G.P. is 3 or –3** **Ans.**

10 If the 4th, 7th and 10th terms of a G.P. are a, b and c respectively; prove that : $b^2 = ac$.

Solution :

Let for the given G.P., first term = A and common ratio R.

Given : 4th term = a $\Rightarrow$ $AR^3 = a$

7th term = b $\Rightarrow$ $AR^6 = b$

and, 10th term = c $\Rightarrow$ $AR^9 = c$

$ac = AR^3 \times AR^9$

$= A^2R^{12} = (AR^6)^2 = b^2$

$\therefore \quad b^2 = ac$ **Hence proved.**

EXERCISE 11(B)

1. Which term of the G.P. :
$-10, \frac{5}{\sqrt{3}}, -\frac{5}{6}, \ldots\ldots\ldots$ is $-\frac{5}{72}$?
2. The fifth term of a G.P. is 81 and its second term is 24. Find the geometric progression.
3. Fourth and seventh terms of a G.P. are $\frac{1}{18}$ and $-\frac{1}{486}$ respectively. Find the G.P.
4. If the first and the third terms of a G.P. are 2 and 8 respectively, find its second term.
5. The product of 3rd and 8th terms of a G.P. is 243. If its 4th term is 3, find its 7th term.

Given : $ar^2 \times ar^7 = 243 \Rightarrow a^2r^9 = 243$

Also, given that : $ar^3 = 3$

$ar^3 = 3 \quad \Rightarrow \quad a = \frac{3}{r^3}$

Now $\quad a^2r^9 = 243 \Rightarrow \left(\frac{3}{r^3}\right)^2 \times r^9 = 243$

i.e. $\quad \frac{9}{r^6} \times r^9 = 243 \Rightarrow r^3 = \frac{243}{9} = 27$

$\Rightarrow r = 3$

$\therefore \quad a = \frac{3}{r^3} = \frac{3}{3^3} = \frac{3}{27} = \frac{1}{9}$

6. Find the geometric progression with 4th term = 54 and 7th term = 1458.
7. Second term of a geometric progression is 6 and its fifth term is 9 times of its third term. Find the geometric progression. Consider that each term of the G.P. is positive.
8. The fourth term, the seventh term and the last term of a geometric progression are 10, 80 and 2560 respectively. Find its first term, common ratio and number of terms.

9. If the 4^{th} and 9^{th} terms of a G.P. are 54 and 13122 respectively, find the G.P. Also, find its general term.

10. The fifth, eight and eleventh terms of a geometric progression are p, q and r respectively. Show that : $q^2 = pr$.

11 Find the 4^{th} term from the end of the series 8, 4, 2, ..., $\frac{1}{128}$.

Solution :

If for the given G.P. common ratio = r and last term = l;

its n^{th} term from the end = $\frac{l}{r^{n-1}}$

According to the given series : $\frac{4}{8} = \frac{1}{2}$, $\frac{2}{4} = \frac{1}{2}$ $\Rightarrow$ $\frac{4}{8} = \frac{2}{4}$

So, given series as a G.P. with common ratio $r = \frac{1}{2}$

Here, $r = \frac{1}{2}$ and last term $l = \frac{1}{128}$

$\Rightarrow$ **4^{th} term from the end** = $\frac{l}{r^{4-1}}$ $\left[\because n^{th} \text{ term from the end} = \frac{l}{r^{n-1}}\right]$

$$= \frac{\frac{1}{128}}{\left(\frac{1}{2}\right)^3} = \frac{1}{128} \times \frac{8}{1} = \frac{1}{16}$$ **Ans.**

Let the given G.P. = a, ar, ar^2, ar^3, ...

$= a, ar, ar^2, ar^3, ..., \frac{l}{r^3}, \frac{l}{r^2}, \frac{l}{r}, l$

$\therefore$ n^{th} term from the end of G.P.

$a, ar, ar^2, ar^3, ..., \frac{l}{r^3}, \frac{l}{r^2}, \frac{l}{r}, l$

= n^{th} term from the beginning of the G.P.

$l, \frac{l}{r}, \frac{l}{r^2}, \frac{l}{r^3}$ ...., ar^2, ar, a

$$= l\left(\frac{1}{r}\right)^{n-1} = \frac{l}{r^{n-1}}$$

12 Find the fifth term from the end of the series 243, 81, 27, 9, ..., $\frac{1}{729}$.

Solution :

$\because$ $\frac{81}{243} = \frac{1}{3}$, $\frac{27}{81} = \frac{1}{3}$, $\frac{9}{27} = \frac{1}{3}$

$\Rightarrow$ Given series is a geometric progression, whose common ratio $r = \frac{1}{3}$ and last term $l = \frac{1}{729}$.

$\Rightarrow$ **5th term from the end** $= \frac{l}{r^{n-1}}$ [$n = 5$]

$$= \frac{\frac{1}{729}}{\left(\frac{1}{3}\right)^4} = \mathbf{\frac{1}{9}}$$ **Ans.**

13 Show that in a G.P., the product of 8th term from its beginning and 8th term from its end is equal to the product of its first and the last terms.

Solution :

Let the G.P. be $a + ar + ar^2 + ar^3 + \ldots\ldots\ldots\ldots + l$

$\therefore$ 8th term from the beginning $= ar^7$

and, 8th term from the last $= \frac{l}{r^{n-1}}$ [$n = 8$]

$$= \frac{l}{r^7}$$

$\therefore$ **8th term from the beginning × 8th term from the end**

$$= ar^7 \times \frac{l}{r^7}$$

$= a \times l =$ **product of the first and the last terms.**

14 If for a G.P., its p^{th}, q^{th} and r^{th} terms are a, b and c respectively; prove that : $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$.

Solution :

Let the first term of the G.P. = A and its common ratio = R

$\therefore$ p^{th} term $= a \Rightarrow AR^{p-1} = a$,

q^{th} term $= b \Rightarrow AR^{q-1} = b$

and, r^{th} term $= c \Rightarrow AR^{r-1} = c$

$\therefore$ $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = (AR^{p-1})^{q-r} \cdot (AR^{q-1})^{r-p} \cdot (AR^{r-1})^{p-q}$

$= A^{q-r} \cdot R^{(p-1)(q-r)} \cdot A^{r-p} R^{(q-1)(r-p)} \cdot A^{p-q} \cdot R^{(r-1)(p-q)}$

$= A^{q-r+r-p+p-q} \cdot R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$

$= A^{\circ} \cdot R^{\circ} = \mathbf{1}$ **Hence Proved.**

15 If a, b and c are in A.P. whereas x, y and z are in G.P. :
Prove that : $x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1$.

Solution :

a, b and c are in A.P. $\Rightarrow$ $2b = a + c$

Also, for x, y and z in G.P., if common ratio is R, $y = x\text{R}$ and $z = x\text{R}^2$

$\therefore \quad x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = x^{b-c} \cdot (x\text{R})^{c-a} \cdot (x\text{R}^2)^{a-b}$

$= x^{b-c} \cdot x^{c-a} \cdot \text{R}^{c-a} \cdot x^{a-b} \cdot \text{R}^{2a-2b}$

$= x^{\circ} \cdot \text{R}^{c+a-2b}$

$= x^{\circ}\, \text{R}^{2b-2b}$ $\quad [\because 2b = a + c]$

$= x^{\circ}\, \text{R}^{\circ} = 1$ **Hence Proved.**

16 If p^{th}, q^{th}, r^{th} and s^{th} terms of a G.P. are in G.P., prove that : $(p - q)$, $(q - r)$ and $(r - s)$ are also in G.P.

Solution :

Let for the given G.P., first term = A and common ratio = R

$\therefore t_p = \text{AR}^{p-1}$, $t_q = \text{AR}^{q-1}$, $t_r = \text{AR}^{r-1}$ and $t_s = \text{AR}^{s-1}$ are in G.P.

$\Rightarrow \dfrac{\text{AR}^{q-1}}{\text{AR}^{p-1}} = \dfrac{\text{AR}^{r-1}}{\text{AR}^{q-1}} = \dfrac{\text{AR}^{s-1}}{\text{AR}^{r-1}}$

$\Rightarrow R^{q-p} = \text{R}^{r-q} = \text{R}^{s-r}$

$\Rightarrow q - p = r - q = s - r$

$\Rightarrow q - p = r - q$ and $r - q = s - r$

$\Rightarrow \dfrac{q-p}{r-q} = 1$ and $\dfrac{r-q}{s-r} = 1$

$\Rightarrow \dfrac{q-p}{r-q} = \dfrac{r-q}{s-r} \Rightarrow (r-q)^2 = (q-p)(s-r)$

$\Rightarrow (q-r)^2 = (p-q)(r-s)$

$\Rightarrow$ **$p - q$, $q - r$ and $r - s$ are in G.P.** **Hence Proved.**

EXERCISE 11(C)

1. Find the seventh term from the end of the series : $\sqrt{2}$, 2, $2\sqrt{2}$, ..., 32.

2. Find the third term from the end of the G.P. $\dfrac{2}{27}, \dfrac{2}{9}, \dfrac{2}{3}$, ..., 162.

3. For the G.P. $\dfrac{1}{27}, \dfrac{1}{9}, \dfrac{1}{3}$, ..., 81;

   find the product of fourth term from the beginning and the fourth term from the end.

4. If for a G.P., p^{th}, q^{th} and r^{th} terms are a, b and c respectively; prove that :

   $(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$

   Proceed as example 15 to show :
   $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$
   Now take log of both the sides.

5. The $(p + q)^{\text{th}}$ term of a G.P. is m and its $(p - q)^{\text{th}}$ term is n, show that :

   its p^{th} term $= \sqrt{mn}$

$ar^{p+q-1} = m$ and $ar^{p-q-1} = n$

$\Rightarrow ar^{p+q-1} \cdot ar^{p-q-1} = mn$

$\Rightarrow a^2r^{2p-2} = mn$ i.e. $(ar^{p-1})^2 = mn$

$\Rightarrow ar^{p-1} = \sqrt{mn}$ i.e. $t_p = \sqrt{mn}$

6. The first term of a G.P. is a and its n^{th} term is b, where n is an even number. If the product of first n numbers of this G.P. is P; prove that :

$$P^2 = (ab)^n.$$

Given : $b = n^{th}$ term $= ar^{n-1}$

P = product of first n terms of the given G.P.

$P = a \times ar \times ar^2 \times ar^3 \times \ldots\ldots\ldots \times b$

$= a \times ar \times ar^2 \times ar^3 \times \ldots\ldots \frac{b}{r^2} \times \frac{b}{r} \times b$

$= (ab) \times (ar \times \frac{b}{r}) \times (ar^2 \times \frac{b}{r^2}) \times \ldots\ldots \frac{n}{2}$ terms

$= (ab) \times (ab) \times (ab) \times \ldots\ldots\ldots \frac{n}{2}$ terms

$= (ab)^{\frac{n}{2}}$

$\therefore$ $\mathbf{P^2 = (ab)^n}$

7. If a, b, c and d are consecutive terms of a G.P.; prove that :

$(a^2 + b^2)$, $(b^2 + c^2)$ and $(c^2 + d^2)$ are in G.P.

Show that : $\frac{b^2+c^2}{a^2+b^2} = \frac{c^2+d^2}{b^2+c^2}$

i.e. $(b^2 + c^2)^2 = (a^2 + b^2)(c^2 + d^2)$

8. If a, b, c and d are consecutive terms of a G.P.; prove that :

$\frac{1}{a^2+b^2}$, $\frac{1}{b^2+c^2}$ and $\frac{1}{c^2+d^2}$ are in G.P.

11.4 Properties of geometric progression :

1. The ratio between the consecutive terms of a G.P. is always the same.

   i.e. if t_1, t_2, t_3, t_4, ... form a G.P.

   $\Rightarrow \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \ldots\ldots\ldots\ldots$

2. In a G.P., the product of the two terms equidistant from its beginning and from its end is always constant which is equal to the product of its first term and its last term.

   i.e. r^{th} term from the beginning × r^{th} term from the end [For every G.P.]

   = constant = first term of the G.P. × its last term

   $\Rightarrow$ 7^{th} term from the beginning × 7^{th} term from the end

   = constant = first term of the G.P. × its last term

3. If a, b and c are in G.P.

   $\Rightarrow \frac{b}{a} = \frac{c}{b}$ i.e. $\mathbf{b^2 = ac}$

   $\therefore$ a, b and c are in G.P. $\Rightarrow$ $b^2 = ac$

   Conversely, $b^2 = ac \Rightarrow a$, b and c are in G.P.

4. In a G.P., if the terms at equal distances are taken, these terms are also in G.P.

   e.g., for G.P. 2, 6, 18, 54, 162, 486, ...

   (i) 2, 18, 162, ... are in G.P.

   (ii) 6, 54, 486, ... are in G.P.

5. If each term of a G.P. be multiplied or divided by the same non-zero number, the resulting series is also a G.P.

6. The series obtained by taking the reciprocals of the terms of a G.P., is also a G.P.

   e.g. 6, 24, 96, 384, ... is a G.P.

   $\Rightarrow \frac{1}{6}, \frac{1}{24}, \frac{1}{96}, \frac{1}{384}$ is also a G.P.

7. If each term of a G.P. is raised to the same non-zero number, the resulting series is also a G.P.

   e.g. 2, 8, 32, ... is a G.P.

   $\Rightarrow$ $2^5, 8^5, 32^5$, ... is also a G.P.

8. (i) If the corresponding terms of two different G.P.s are multiplied together; the resulting series, so obtained. is also a G.P.

   e.g. a, ar, ar^2, ar^3, ... is a G.P.

   and, A, AR, AR^2, AR^3, ... is a G.P.

   then, $a \times A$, $(ar) \times (AR)$, $(ar^2) \times (AR^2)$, ... is also a G.P.

   (ii) In the same way, if the terms of a G.P. be divided by corresponding terms of some other G.P., the resulting series, so obtained, is also a G.P.

   Thus for G.P. a, ar, ar^2, ar^3, ...

   and for G.P. A, AR, AR^2, AR^3, ...

   $\frac{a}{A}, \frac{ar}{AR}, \frac{ar^2}{AR^2}$, ... is also a G.P.

17 If a, b and c are in G.P., show that $\frac{1}{a+b}, \frac{1}{2b}$ and $\frac{1}{b+c}$ are in A.P.

Solution :

$\frac{1}{a+b}, \frac{1}{2b}$ and $\frac{1}{b+c}$ will be in A.P.

if $2 \times \frac{1}{2b} = \frac{1}{a+b} + \frac{1}{b+c}$ *i.e.* $\frac{1}{b} = \frac{1}{a+b} + \frac{1}{b+c}$

Let common ratio of the G.P. = r.

$\therefore$ $b = ar$, $c = br = (ar)r = ar^2$

$$\frac{1}{a+b} + \frac{1}{b+c} = \frac{1}{a+ar} + \frac{1}{ar+ar^2}$$

$$= \frac{1}{a(1+r)} + \frac{1}{ar(1+r)}$$

$$= \frac{r+1}{ar(1+r)} = \frac{1}{ar} = \frac{1}{b}$$

$\therefore$ $\frac{1}{a+b}, \frac{1}{2b}$ and $\frac{1}{b+c}$, are in A.P. **Hence Proved.**

EXERCISE 11(D)

1. If a, b and c are in G.P., prove that :
   (i) log a, log b and log c are in A.P.
   (ii) log a^n, log b^n and log c^n are in A.P.

2. If $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ and a, b, c are in G.P.; prove that : x, y, z are in A.P.

Let $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$

$\Rightarrow a^{\frac{1}{x}} = k$, $b^{\frac{1}{y}} = k$ and $c^{\frac{1}{z}} = k$

$\Rightarrow a = k^x$, $b = k^y$ and $c = k^z$

a, b, c are in G.P. $\Rightarrow b^2 = ac$

$\Rightarrow (k^y)^2 = k^x \times k^z$ i.e. $2y = x + z$

$\Rightarrow x$, y, z are in A.P.

3. If each term of a G.P. is raised to the power x, show that the resulting sequence is also a G.P.

4. If a, b and c are in A.P, a, x, b are in G.P. whereas b, y and c are also in G.P.
   Show that : x^2, b^2, y^2 are in A.P.

a, b, c in A.P. $\Rightarrow 2b = a + c$,

a, x, b in G.P. $\Rightarrow x^2 = ab$

and b, y, c are in G.P. $\Rightarrow y^2 = bc$

$x^2 + y^2 = ab + bc$

$= b(a + c) = b \times 2b = 2b^2$

$\therefore$ x^2, b^2 and y^2 are in A.P.

5. If a, b, c are in G.P. and a, x, b, y, c are in A.P., prove that :
   (i) $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$ (ii) $\frac{a}{x} + \frac{c}{y} = 2$.

6. If $\log_x a^n$, $\log_x b^n$ and $\log_x c^n$ are in A.P., prove that a, b, c are in G.P.

7. If a, b and c are in A.P. and also in G.P., show that : $a = b = c$.

$2b = a + c$ and $b^2 = ac \Rightarrow \left(\frac{a+c}{2}\right)^2 = ac$

On simplifying it, will give $a = c$

and then $2b = a + c = a + a = 2a \Rightarrow b = a$

11.5 Sum of n terms of a G.P. :

If for a G.P., the first term = a, the common ratio = r and number of terms = n, the sum of n terms is

$$S_n = a + ar + ar^2 + ar^3 + \text{...........} + ar^{n-1} \qquad \text{...........} \text{ I}$$

Case I : When $r = 1$

$$S_n = a + a + a + a + \text{..............} \text{ to } n \text{ terms}$$
$$= na$$

Case II : When $r < 1$

$$S_n = a + ar + ar^2 + \text{..............} + ar^{n-1} \qquad \text{..........} \text{ I}$$
$$r \times S_n = ar + ar^2 + \text{..............} + ar^{n-1} + ar^n \qquad \text{..........} \text{ II}$$

Equation I – equation II, gives :

$$S_n - r \times S_n = a - ar^n$$

$\Rightarrow$ $S_n(1 - r) = a(1 - r^n)$

i.e. $$S_n = \frac{a(1-r^n)}{1-r}$$

Case III : When $r > 1$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$$

Thus : $S_n = na$, when $r = 1$

$= \dfrac{a(1-r^n)}{1-r}$, when $r < 1$

$= \dfrac{a(r^n-1)}{r-1}$, when $r > 1$

Further, if l = last term of the G.P.

$= ar^{n-1}$

$\Rightarrow \quad lr = ar^{n-1} \times r = ar^n$

$\therefore$ For $r < 1$, we have

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a-ar^n}{1-r} = \frac{a-lr}{1-r} \qquad [\because ar^n = lr]$$

And, for r > 1, we have

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{ar^n-a}{r-1} = \frac{lr-a}{r-1}$$

For a G.P. with first term = a and common ratio = r

(i) if $r = 1$, $S_n = na$ (ii) if $r < 1$, $S = \dfrac{a(1-r^n)}{1-r} = \dfrac{a-lr}{1-r}$ and

(iii) if $r > 1$, $S_n = \dfrac{a(r^n-1)}{r-1} = \dfrac{lr-a}{r-1}$

18 Find the sum of 10 terms of the series : 96 - 48 + 24 ....

Solution :

$\because -\dfrac{48}{96} = -\dfrac{1}{2}, \dfrac{24}{-48} = -\dfrac{1}{2} = \text{..............} =$ a constant

$\therefore$ Given series is a G.P. with first term $a = 96$

and common ratio $r = -\dfrac{1}{2}$ (clearly, $r < 1$)

Now, $S_n = \dfrac{a(1-r^n)}{1-r}$

$$\Rightarrow \quad S_{10} = \frac{96\left[1-\left(-\frac{1}{2}\right)^{10}\right]}{1-\left(-\frac{1}{2}\right)} = 96 \times \frac{2}{3} \times \left[1-\frac{1}{1024}\right]$$

$$= 96 \times \frac{2}{3} \times \frac{1023}{1024} = \frac{1023}{16} = 63\frac{15}{16}$$ **Ans.**

19 Find the sum of 8 terms of the G.P. : 3 + 6 + 12 + 24 + ...

Solution :

Here, first term $a = 3$, common ratio $r = \frac{6}{3} = 2$ and number of terms (n) to be added = 8

$$\therefore \quad S_n = \frac{a(r^n - 1)}{r - 1} \qquad [\because r > 1]$$

$$\Rightarrow \quad S_8 = \frac{3(2^8 - 1)}{2 - 1} = \frac{3 \times (256 - 1)}{1} = 765$$ **Ans.**

20 Find the sum of the geometric series : $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$, ... upto 12 terms.

Solution :

Clearly, $a = 1$, $r = \frac{1}{2}$ and $n = 12$

$$\therefore \quad S_n = \frac{a(1 - r^n)}{1 - r} \qquad [\because r < 1]$$

$$\Rightarrow S_{12} = \frac{1 \times \left[1 - \left(\frac{1}{2}\right)^{12}\right]}{1 - \frac{1}{2}}$$

$$= 2 \times \left[1 - \frac{1}{4096}\right] = 2 \times \frac{4095}{4096} = \frac{4095}{2048} = 1\frac{2047}{2048}$$ **Ans.**

21 Find the sum of 10 terms of the geometric progression : $1 + \sqrt{3} + 3 + 3\sqrt{3} +$ ....

Solution :

Here, $a = 1$, $r = \frac{\sqrt{3}}{1} = \sqrt{3}$ and $n = 10$

$$\therefore \quad S_n = \frac{a(r^n - 1)}{r - 1} \qquad [\because r > 1]$$

$$\Rightarrow \quad S_{10} = \frac{1 \times \left[(\sqrt{3})^{10} - 1\right]}{\sqrt{3} - 1}$$

$$= \frac{3^5 - 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{243 - 1}{3 - 1} \times (\sqrt{3} + 1)$$

$$= \frac{242}{2} \times (\sqrt{3} + 1) = 121(\sqrt{3} + 1)$$ **Ans.**

22 How many terms of the G.P.

$\frac{2}{9}, -\frac{1}{3}, \frac{1}{2}$, ... must be added to get the sum equal to $\frac{55}{72}$?

Solution :

Here, $a = \frac{2}{9}$, $r = \frac{-\frac{1}{3}}{\frac{2}{9}} = -\frac{1}{3} \times \frac{9}{2} = -\frac{3}{2}$ and $S_n = \frac{55}{72}$

$\therefore \quad S_n = \frac{a(1-r^n)}{1-r}$ $\qquad [\because r < 1]$

$\Rightarrow \quad \frac{55}{72} = \frac{\frac{2}{9}\times\left[1-\left(\frac{-3}{2}\right)^n\right]}{1-\left(\frac{-3}{2}\right)}$

$\Rightarrow \quad \frac{55}{72} \times \frac{5}{2} \times \frac{9}{2} = 1 - \left(-\frac{3}{2}\right)^n$ *i.e.* $\frac{275}{32} = 1 - \left(-\frac{3}{2}\right)^n$

$\Rightarrow \quad \left(-\frac{3}{2}\right)^n = 1 - \frac{275}{32} = -\frac{243}{32} = \left(-\frac{3}{2}\right)^5$

$\Rightarrow \quad n = 5$

$\therefore$ **Required number of terms = 5** **Ans.**

23 Find the sum of the G.P. : 2 + 6 + 18 + 54 + ... + 4374.

Solution :

Here, $a = 2$, $r = \frac{6}{2} = 3$ and $l = 4374$; clearly $r > 0$.

$\therefore \quad S = \frac{lr-a}{r-1} = \frac{4374\times3-2}{3-1} = 6560$ **Ans.**

Alternative method :

Let the number of terms in the given G.P. = n

$\therefore \quad ar^{n-1} = 4374 \quad \Rightarrow 2 \times 3^{n-1} = 4374$

i.e. $3^{n-1} = \frac{4374}{2} = 2187 = 3^7$ $\qquad [2187 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3]$

i.e. $n - 1 = 7$ and $n = 8$

Now sum of the sequence $= \frac{a(r^n-1)}{r-1}$ $\qquad [\because r > 1]$

$= \frac{2\times(3^8-1)}{3-1} = 6560$ **Ans.**

24 A G.P. has first term $a = 3$, last term $l = 96$ and sum of n terms $S = 189$. Find the number of terms in it.

Solution :

$$S = \frac{lr - a}{r - 1} \quad \Rightarrow \quad 189 = \frac{96 \times r - 3}{r - 1}$$

On solving, it gives $r = 2$

Now, $l = ar^{n-1} \quad \Rightarrow \quad 96 = 3 \times 2^{n-1}$

i.e. $\frac{96}{3} = 2^{n-1} \quad \Rightarrow \quad 32 = 2^{n-1}$,

i.e. $2^5 = 2^{n-1} \quad \Rightarrow \quad n - 1 = 5$ and $\mathbf{n = 6}$ **Ans.**

25 Find the sum of first n terms of the series : 5 + 55 + 555 + ...

Solution :

Required sum = 5 + 55 + 555 + ... upto n terms

= 5[1 + 11 + 111 + ... upto n terms]

$= \frac{5}{9}$(9 + 99 + 999 + ... upto n terms)

$= \frac{5}{9}$[(10 – 1) + (100 – 1) + (1000 – 1) + ... upto n terms]

$= \frac{5}{9}$[(10 + 10^2 + 10^3 + ... upto n terms)

– (1 + 1 + 1 + ... upto n terms)]

$$= \frac{5}{9} \times \left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{5}{9} \times \frac{10}{9}(10^n - 1) - \frac{5}{9}n$$

$$= \mathbf{\frac{50}{81}(10^n - 1) - \frac{5}{9}n}$$ **Ans.**

(i) 3 + 33 + 333 + ... upto n terms

= 3[1 + 11 + 111 + ... upto n terms]

$= \frac{3}{9}$[9 + 99 + 999 + ... upto n terms]

(ii) 7 + 77 + 777 + ... upto n terms

= 7[1 + 11 + 111 + ... upto n terms]

$= \frac{7}{9}$[9 + 99 + 999 + ... upto n terms] and so on.

26 Find the sum of the series :

0·7 + 0·77 + 0·777 + ... upto n terms.

Solution :

Required sum = 0·7 + 0·77 + 0·777 + ... upto n terms

$= 7[0{\cdot}1 + 0{\cdot}11 + 0{\cdot}111 + \ldots \text{ upto } n \text{ terms}]$

$= \frac{7}{9}(0{\cdot}9 + 0{\cdot}99 + 0{\cdot}999 + \ldots \text{ upto } n \text{ terms})$

$= \frac{7}{9}[(1 - 0{\cdot}1) + (1 - 0{\cdot}01) + (1 - 0{\cdot}001) + \ldots \text{ upto } n \text{ terms}]$

$= \frac{7}{9}[(1 + 1 + 1 + \ldots \text{ upto } n \text{ terms})$

$- (0{\cdot}1 + 0{\cdot}01 + 0{\cdot}001 + \ldots \text{ upto } n \text{ terms})]$

$= \frac{7}{9}[(1 + 1 + 1 + \ldots \text{ upto } n \text{ terms})]$

$- (\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots \text{ upto } n \text{ terms})$

$$= \frac{7}{9}\left[n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right] \qquad \left[\because r = \frac{1}{10} \text{ so } r < 1\right]$$

$$= \frac{7}{9}n - \frac{7}{9} \times \frac{10}{9} \times \frac{1}{10}\left(1-\frac{1}{10^n}\right)$$

$$= \frac{7n}{9} - \frac{7}{81}\left(1-\frac{1}{10^n}\right)$$ **Ans.**

(i) 0·4 + 0·44 + 0·444 + ... upto n terms

$= \frac{4}{9}[0{\cdot}9 + 0{\cdot}99 + 0{\cdot}999 + \ldots \text{ upto } n \text{ terms}]$

(ii) 0·5 + 0·55 + 0·555 + ... upto n terms

$= \frac{5}{9}[0{\cdot}9 + 0{\cdot}99 + 0{\cdot}999 + \ldots \text{ upto } n \text{ terms}]$ and so on.

EXERCISE 11(E)

1. Find the sum of G.P. :
   (i) $1 + 3 + 9 + 27 + \dots\dots$ to 12 terms.
   (ii) $0{\cdot}3 + 0{\cdot}03 + 0{\cdot}003 + 0{\cdot}0003 + \dots\dots$ to 8 terms.
   (iii) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots\dots$ to 9 terms.
   (iv) $1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots\dots$ to n terms.
   (v) $\frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \dots\dots$ upto n terms.
   (vi) $\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \dots\dots$ to n terms.
2. How many terms of the geometric progression $1 + 4 + 16 + 64 + \dots\dots$ must be added to get sum equal to 5461 ?
3. The first term of a G.P. is 27 and its 8th term is $\frac{1}{81}$. Find the sum of its first 10 terms.
4. A boy spends ₹ 10 on first day, ₹ 20 on second day, ₹ 40 on third day and so on. Find how much, in all, will he spend in 12 days ?

   G.P. formed is : ₹ 10 + ₹ 20 + ₹ 40 + ...

5. The 4th and the 7th terms of a G.P. are $\frac{1}{27}$ and $\frac{1}{729}$ respectively. Find the sum of n terms of this G.P.
6. A geometric progression has common ratio = 3 and last term = 486. If the sum of its terms is 728; find its first term.
7. Find the sum of G.P. : 3, 6, 12, ..., 1536.
8. How many terms of the series $2 + 6 + 18 + \dots\dots$ must be taken to make the sum equal to 728 ?
9. In a G.P., the ratio between the sum of first three terms and that of the first six terms is 125 : 152.
   Find its common ratio.

   Given : $\frac{a(r^3-1)}{r-1} : \frac{a(r^6-1)}{r-1} = 125 : 152.$

10. Find the sum of n terms of the series :
    (i) $4 + 44 + 444 + \dots\dots$.
    (ii) $0{\cdot}8 + 0{\cdot}88 + 0{\cdot}888 + \dots\dots$.
    (iii) $2 + 22 + 222 + \dots\dots$.
    (iv) $0{\cdot}5 + 0{\cdot}55 + 0{\cdot}555 + \dots\dots$.
11. Find how many terms of G.P. $\frac{2}{9} - \frac{1}{3} + \frac{1}{2} \dots\dots$ must be added to get the sum equal to $\frac{55}{72}$?
12. If the sum of $1 + 2 + 2^2 + \dots\dots + 2^{n-1}$ is 255, find the value of n.

11.6 Sum of Infinite Terms of a G.P. :

Consider a geometric progression with first term = a and common ratio = r such that the numerical value of r is less than 1 (one).

i.e. $|r| < 1 \Rightarrow -1 < r < 1$

Sum of infinite terms in G.P. = $\frac{a}{1-r}$, if $|r| < 1$

27 Find the sum of G.P. : $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \dots\dots$ upto infinite terms.

Solution :

Since, first term $(a) = 1$, common ratio $(r) = \dfrac{-\frac{1}{4}}{1} = -\dfrac{1}{4}$ and $|r| = \left|-\dfrac{1}{4}\right| = \dfrac{1}{4}$ which is less than 1 (one).

$\therefore$ **Required sum** $= \dfrac{a}{1-r} = \dfrac{1}{1-\left(-\frac{1}{4}\right)} = \mathbf{\dfrac{4}{5}}$ **Ans.**

28 Find the first term of the G.P. with second term = 2 and sum of infinite terms = 8.

Solution :

Given : $ar = 2$ and $\dfrac{a}{1-r} = 8$ $[\because |r| < 1]$

$\Rightarrow$ $a = \dfrac{2}{r}$ and $a = 8 - 8r$

$\Rightarrow$ $\dfrac{2}{r} = 8 - 8r$ *i.e.* $2 = 8r - 8r^2$

$\Rightarrow$ $4r^2 - 4r + 1 = 0$ *i.e.* $(2r - 1)^2 = 0$

$\Rightarrow$ $2r - 1 = 0$ *i.e.* $r = \dfrac{1}{2}$

Now, $ar = 2$ $\Rightarrow$ $a \times \dfrac{1}{2} = 2$ *i.e.* $a = 4$

$\therefore$ **First term of the given G.P. is 4** **Ans.**

11.7 Recurring decimals :

In order to find the value of a recurring decimal, we use the formula :

$$S = \frac{a}{1-r}, \text{ where } |r| < 1.$$

29 Evaluate : $0{\cdot}4\overline{37}$.

Solution :

$$0{\cdot}4\overline{37} = 0{\cdot}4373737\ldots\ldots \text{ upto infinity}$$

$$= 0{\cdot}4 + 0{\cdot}037 + 0{\cdot}00037 + 0{\cdot}0000037 + \ldots\ldots \text{ upto infinity}$$

$$= \frac{4}{10} + \frac{37}{1000} + \frac{37}{100000} + \frac{37}{10000000} + \ldots\ldots \text{ upto infinity}$$

$$= \frac{4}{10} + \frac{37}{1000}\left[1 + \frac{1}{100} + \frac{1}{10000} + \ldots\ldots \text{upto infinity}\right]$$

$$= \frac{4}{10} + \frac{37}{1000}\left[\frac{1}{1-\frac{1}{100}}\right]$$

$$= \frac{4}{10} + \frac{37}{1000} \times \frac{100}{100-1}$$

$$= \frac{4}{10} + \frac{37}{990} = \frac{396+37}{990} = \mathbf{\frac{433}{990}}$$ **Ans.**

30 Find the sum of the infinite terms in the following geometric progression :
$(\sqrt{2}+1) + 1 + (\sqrt{2}-1) + \ldots\ldots\ldots\ldots\ldots\ldots$.

Solution :

$a = \sqrt{2} + 1$ and $r = \frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \sqrt{2} - 1$, which is less than 1.

$\therefore$ **Sum of its infinite G.P.** $= \frac{a}{1-r} = \frac{\sqrt{2}+1}{1-\sqrt{2}+1}$

$$= \frac{\sqrt{2}+1}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}}$$

$$= \frac{2\sqrt{2}+2+2+\sqrt{2}}{4-2} = \mathbf{\frac{4+3\sqrt{2}}{2}}$$ **Ans.**

31 Find the common ratio of an infinite G.P., whose each term is ten times the sum of its succeeding terms.

Solution :

If first term = a and common ratio = r

Given : $a = 10(ar + ar^2 + ar^3 + \ldots\ldots\ldots\ldots)$

$\Rightarrow$ $a = 10 \times \frac{ar}{1-r} \Rightarrow \mathbf{r = \frac{1}{11}}$ **Ans.**

EXERCISE 11(F)

1. Find the sum of infinite terms of each of the following geometric progression :

   (i) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\ldots\ldots\ldots$.

   (ii) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots\ldots\ldots\ldots$.

   (iii) $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\ldots\ldots\ldots$.

   (iv) $\sqrt{2} - \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{1}{4\sqrt{2}} + \ldots\ldots\ldots\ldots$.

   (v) $\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + \frac{1}{9\sqrt{3}} + \ldots\ldots\ldots\ldots$.

2. The second term of a G.P. is 9 and sum of its infinite terms is 48. Find its first three terms.

3. Evaluate :

   (i) $0{\cdot}\dot{2}$ (ii) $0{\cdot}5\dot{2}$

   (iii) $0{\cdot}4\overline{23}$ (iv) $0{\cdot}7\dot{6}\dot{2}$

   (v) $0{\cdot}02\dot{7}$

4. Find the sum of infinite terms of the series

   $\frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \frac{2}{5^5} + \frac{3}{5^6} + \ldots\ldots\ldots$

   Given series

   $= \left(\frac{2}{5} + \frac{2}{5^3} + \frac{2}{5^5} + \ldots\ldots\right) + \left(\frac{3}{5^2} + \frac{3}{5^4} + \frac{3}{5^6} + \ldots\ldots\ldots\right)$

11.8 Geometric mean between numbers a and b :

If a and b are two positive numbers then a, G, b are in G.P.

$$\Rightarrow \frac{G}{a} = \frac{b}{G} \quad i.e. \quad \boxed{G^2 = ab} \quad \text{and} \quad \boxed{G = \sqrt{ab}}$$

Here, G is said to be a **geometric mean between *a* and *b*.**

32 Find the geometric mean between

(i) 3 and 12 (ii) 3 and 243

Solution :

(i) **Geometric mean between 3 and 12**

$= \sqrt{3\times12} = \sqrt{36} = \mathbf{6}$ **Ans.**

(ii) **Geometric mean between 3 and 243**

$= \sqrt{3\times243} = \sqrt{729} = \mathbf{27}$ **Ans.**

11.9 n Geometric means between a and b :

33 Insert two numbers between 3 and 81 so that all the four numbers are in G.P.

Solution :

Let the required numbers be G_1 and G_2

$\Rightarrow$ 3, G_1, G_2 and 81 are in G.P.

If common ratio be r

$\therefore$ 81 = 4[th] term of the G.P.

$\Rightarrow$ $81 = ar^3$ i.e. $81 = 3 \times r^3$

$\Rightarrow$ $r^3 = 27$ and $r = 3$

$\therefore$ $G_1 = ar = 3 \times 3 = \mathbf{9}$ and $G_2 = ar^2 = 3 \times 3^2 = \mathbf{27}$ **Ans.**

34 Insert three numbers between 1 and 256 so that the sequence is a G.P.

Solution :

Let the required numbers be G_1, G_2 and G_3

$\Rightarrow$ 1, G_1, G_2, G_3 and 256 are in G.P.

$\Rightarrow$ 256 = 5[th] term of the G.P.

$= 1 \times r^4$ [r = common ratio]

$\Rightarrow$ $r^4 = 256 = 4^4$ i.e. $r = 4$

$\therefore$ $G_1 = 1 \times r = 1 \times 4 = \mathbf{4}$, $G_2 = 1 \times r^2 = 1 \times 4^2 = \mathbf{16}$

$G_3 = 1 \times r^3 = 1 \times 4^3 = \mathbf{64}$ **Ans.**

35 The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is -1. Find the terms.

Solution :

When the product of terms in G.P. is given :

(i) take terms as : $\frac{a}{r}$, a, ar, if the number of terms is three,

(ii) take terms as : $\frac{a}{r^3}$, $\frac{a}{r}$, a, ar^3, if the number of terms is four,

(iii) take terms as : $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar^2, if the number of terms is five and so on.

Let the terms be $\frac{a}{r}$, a and ar

$\Rightarrow \quad \frac{a}{r} \times a \times ar = -1 \quad \Rightarrow \quad a^3 = -1$ and $a = -1$

Now, $\quad \frac{a}{r} + a + ar = \frac{13}{12} \quad \Rightarrow \quad \frac{-1}{r} - 1 - r = \frac{13}{12}$

i.e. $\quad 12r^2 + 25r + 12 = 0 \quad \Rightarrow 12r^2 + 16r + 9r + 12 = 0$

i.e. $\quad (3r + 4)(4r + 3) = 0 \quad \Rightarrow r = -\frac{4}{3}$ or $-\frac{3}{4}$

$\therefore$ **The required terms are :**

$$\frac{a}{r}, a, ar = \frac{-1}{-\frac{4}{3}}, -1, -1 \times -\frac{4}{3} \quad \text{or} \quad \frac{-1}{-\frac{3}{4}}, -1, -1 \times -\frac{3}{4}$$

$$= \mathbf{\frac{3}{4}, -1, \frac{4}{3} \quad or \quad \frac{4}{3}, -1, \frac{3}{4}}$$ **Ans.**

36 Find three consecutive terms in G.P. whose sum is 13 and sum of whose squares is 91.

Solution :

Let the required terms in G.P. be a, ar and ar^2

$\therefore \quad a + ar + ar^2 = 13 \quad$ and $\quad a^2 + a^2r^2 + a^2r^4 = 91$

Now, $\quad (a + ar + ar^2)^2 = 13^2$

$\Rightarrow a^2 + a^2r^2 + a^2r^4 + 2 \times a \times ar + 2ar \times ar^2 + 2 \times ar^2 \times a = 169$

$\Rightarrow \quad 91 + 2ar(a + ar^2 + ar) = 169$

$\Rightarrow \quad 2ar \times 13 = 169 - 91 = 78 \quad$ *i.e.* $\quad ar = \frac{78}{2\times13} = 3$

$$ar = 3 \quad \Rightarrow \quad r = \frac{3}{a}$$

Now, $a + ar + ar^2 = 13 \Rightarrow a + 3 + a \times \frac{9}{a^2} = 13$

i.e. $a^2 - 10a + 9 = 0 \Rightarrow a = 1$ or $a = 9$

i.e. $a = 1$ or $a = 9$

$a = 1 \Rightarrow r = \frac{3}{a} = \frac{3}{1} = 3$

$a = 9 \Rightarrow r = \frac{3}{a} = \frac{3}{9} = \frac{1}{3}$

$\therefore$ $a = 1$ and $r = 3$ $\Rightarrow$ **Required terms in G.P.** $= a, ar, ar^2$

$= 1, 1 \times 3, 1 \times 3^2$

$=$ **1, 3, 9** **Ans.**

and, $a = 9$ and $r = \frac{1}{3}$ $\Rightarrow$ **Required terms in G.P.** $= a, ar, ar^2$

$= 9, 9 \times \frac{1}{3}, 9 \times \left(\frac{1}{3}\right)^2$

$=$ **9, 3, 1** **Ans.**

EXERCISE 11(G)

1. Find the geometric mean between :

   (i) $\frac{4}{9}$ and $\frac{9}{4}$ (ii) 14 and $\frac{7}{32}$

   (iii) $2a$ and $8a^3$

2. Find three geometric means between $\frac{1}{3}$ and 432.

3. Find :

   (i) two geometric means between 2 and 16.

   (ii) four geometric means between 3 and 96.

   (iii) five geometric means between $3\frac{5}{9}$ and $40\frac{1}{2}$.

4. The sum of three numbers in G.P. is $\frac{39}{10}$ and their product is 1. Find the numbers.

5. Find three numbers in G.P. whose sum is 52 and the sum of whose product in pairs is 624.

   Given : $a + ar + ar^2 = 52$ and

   $a \times ar + ar \times ar^2 + ar^2 \times a = 624$

   $\Rightarrow ar(a + ar^2 + ar) = 624$

6. The sum of three numbers in G.P. is 21 and the sum of their squares is 189. Find the numbers.

7. If $y = x + x^2 + x^3 + \ldots\ldots\ldots \infty$, where $|x| < 1$, prove that : $x = \frac{y}{1+y}$.

8. Find five geometric means between 1 and 27.

9. The first term of a G.P. is -3 and the square of the second term is equal to its 4^{th} term. Find its 7^{th} term.

10. The sum of an infinite G.P. is $\frac{80}{9}$ and its common ratio $-\frac{4}{5}$. Find its first term.