13 Section and Mid-Point Formula

13.1 Introduction :

For any two known (given) points in a co-ordinate (Cartesian) plane, the knowledge of *co-ordinate geometry* may be used to find :

(i) the distance between the given points,

(ii) the co-ordinates of a point which divides the line joining the given points in a given ratio,

(iii) the co-ordinates of the mid-point of the line segment joining the two given points,

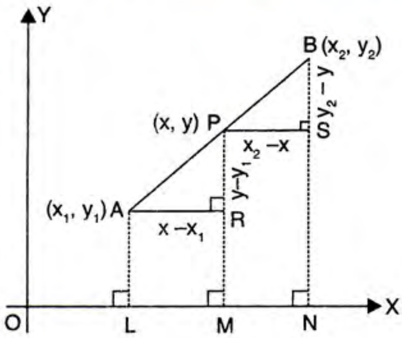
(iv) equation of the straight line through the given points,

(v) equation of the perpendicular bisector of the line segment obtained on joining the given two points, etc.

13.2 The Section Formula :

*To find the co-ordinates of a point which divides the line segment joining two given points **in a given ratio.***

(If a point P lies in a line segment joining the points A and B, then P divides AB in the ratio AP : PB).



Let AB be a line joining the points A = (x_1, y_1) and B = (x_2, y_2) and point P divides the line segment AB in the ratio $m_1 : m_2$.

i.e. $$\frac{AP}{PB} = \frac{m_1}{m_2}$$

Required to find : The co-ordinates of point P.

Let P = (x, y)

Draw AL, PM and BN perpendiculars on the x-axis. Thus, AL, PM and BN are parallel lines. It is clear from the figure that :

$$AR = LM = OM - OL = x - x_1;$$

$$PR = PM - RM = PM - AL = y - y_1;$$

$$PS = MN = ON - OM = x_2 - x$$

and, $$BS = BN - SN = BN - PM = y_2 - y$$

Since, Δ APR and Δ PBS are similar.

$$\therefore \quad \frac{AR}{PS} = \frac{PR}{BS} = \frac{AP}{PB}$$ [Corresponding sides of similar Δs are in proportion]

$$\frac{AR}{PS} = \frac{AP}{PB} \Rightarrow \frac{x - x_1}{x_2 - x} = \frac{m_1}{m_2}$$

$$\Rightarrow m_2x - m_2x_1 = m_1x_2 - m_1x$$ [By cross multiplication]

$$\Rightarrow m_1x + m_2x = m_1x_2 + m_2x_1$$

$$\Rightarrow x(m_1 + m_2) = m_1x_2 + m_2x_1$$

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

Since,

$$\frac{PR}{BS} = \frac{AP}{PB} \Rightarrow \frac{y - y_1}{y_2 - y} = \frac{m_1}{m_2} \Rightarrow y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\therefore \text{ Co-ordinates of P} = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

1 Find the co-ordinates of point P which divides the join of A (4, –5) and B (6, 3) in the ratio 2 : 5.

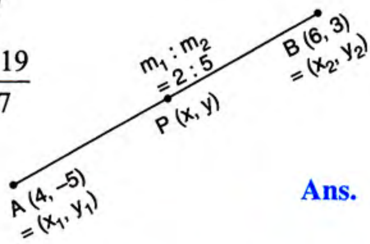
Solution :

Let the co-ordinates of P be (x, y)

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2 \times 6 + 5 \times 4}{2 + 5} = \frac{32}{7}$$

$$\text{and,} \quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2 \times 3 + 5 \times -5}{2 + 5} = \frac{-19}{7}$$

$$\therefore \quad \mathbf{P} = \left(\frac{32}{7}, \frac{-19}{7}\right)$$ **Ans.**



Conversely, ***to find the ratio in which the line joining the two points is divided by a given point.***

2 Find the ratio in which the point (5, 4) divides the line joining points (2, 1) and (7, 6).

Solution :

Let the required ratio be $m_1 : m_2$

Take $(2, 1) = (x_1, y_1)$; $(7, 6) = (x_2, y_2)$ and $(5, 4) = (x, y)$

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \Rightarrow 5 = \frac{m_1 \times 7 + m_2 \times 2}{m_1 + m_2}$$

$$\Rightarrow \quad 5m_1 + 5m_2 = 7m_1 + 2m_2$$
$$\Rightarrow \quad 2m_1 = 3m_2$$
$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{3}{2}$$

$\therefore$ **The required ratio is 3 : 2.** **Ans.**

Alternative method :

In order to find the ratio in which the join of two given points is divided by a third point, take $m_1 : m_2 = k : 1$.

By doing so, two unknowns m_1 and m_2 are reduced to one unknown *i.e.* k and the section formula becomes :

$$x = \frac{kx_2 + x_1}{k+1} \text{ and } y = \frac{ky_2 + y_1}{k+1}$$

$m_1 : m_2$
$= \frac{m_1}{m_2} : \frac{m_2}{m_2}$
$= k : 1$
$\therefore k = \frac{m_1}{m_2}$

Let the required ratio be $k : 1$ $(= m_1 : m_2)$.

$$\therefore \quad x = \frac{kx_2 + x_1}{k+1} \quad \Rightarrow \quad 5 = \frac{k \times 7 + 2}{k+1}$$
$$\Rightarrow \quad 5k + 5 = 7k + 2$$
$$\Rightarrow \quad 2k = 3$$
$$\Rightarrow \quad k = \frac{3}{2}$$

$\therefore$ The required ratio $= k : 1 = \frac{3}{2} : 1 =$ **3 : 2** **Ans.**

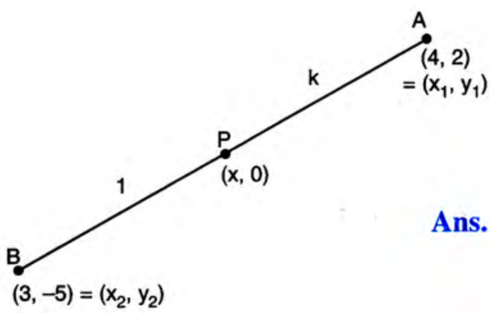
3 In what ratio is the line joining the points (4, 2) and (3, –5) divided by the x-axis ? Also, find the co-ordinates of the point of intersection.

Solution :

Let the required ratio be $k : 1$ and the point on the x-axis be $(x, 0)$.

Since, $\quad y = \frac{ky_2 + y_1}{k+1}$ $\quad$ [Taking $(4, 2) = (x_1, y_1)$ and $(3, -5) = (x_2, y_2)$]

$$\Rightarrow \quad 0 = \frac{k \times -5 + 2}{k+1}$$
$$\Rightarrow \quad 0 = -5k + 2$$
$$\Rightarrow \quad k = \frac{2}{5}$$
$$\Rightarrow \quad m_1 : m_2 = \mathbf{2 : 5}$$ **Ans.**



Now, $\quad x = \frac{2 \times 3 + 5 \times 4}{2+5}$
$$= \frac{26}{7}$$

$\therefore$ **The ratio = 2 : 5 and the required point of intersection =** $\left(\frac{26}{7}, 0\right)$ **Ans.**

4 Calculate the ratio in which the line joining the points (4, 6) and (–5, –4) is divided by the line $y = 3$. Also, find the co-ordinates of the point of intersection.

Solution :

The co-ordinates of every point on the line $y = 3$ will be of the type $(x, 3)$.

Now, $y = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$ [Taking : $(x, 3) = (x, y)$, $(4, 6) = (x_1, y_1)$ and $(-5, -4) = (x_2, y_2)$]

$\Rightarrow \quad 3 = \dfrac{m_1 \times -4 + m_2 \times 6}{m_1 + m_2}$

$\Rightarrow \quad 3m_1 + 3m_2 = -4m_1 + 6m_2 \quad \Rightarrow \quad 7m_1 = 3m_2 \quad \Rightarrow \quad \dfrac{m_1}{m_2} = \dfrac{3}{7}$

$\therefore$ **The required ratio is 3 : 7** **Ans.**

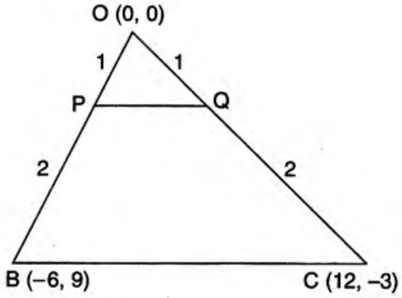
Now, $x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2} \quad \Rightarrow \quad x = \dfrac{3 \times -5 + 7 \times 4}{3 + 7} = \dfrac{13}{10}$

$\therefore$ **The required point of intersection =** $\left(\dfrac{13}{10}, 3\right)$ **Ans.**

5 The origin O, B (–6, 9) and C (12, –3) are vertices of triangle OBC. Point P divides OB in the ratio 1 : 2 and point Q divides OC in the ratio 1 : 2. Find the co-ordinates of points P and Q. Also, show that : $PQ = \dfrac{1}{3}BC$.

Solution :

For point P : $m_1 : m_2 = 1 : 2$, $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (-6, 9)$



$\therefore P = \left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$= \left(\dfrac{1 \times -6 + 2 \times 0}{1 + 2}, \dfrac{1 \times 9 + 2 \times 0}{1 + 2}\right)$

$=$ **(–2, 3)** **Ans.**

For point Q : $m_1 : m_2 = 1 : 2$, $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (12, -3)$

$\therefore \quad Q = \left(\dfrac{1 \times 12 + 2 \times 0}{1 + 2}, \dfrac{1 \times -3 + 2 \times 0}{1 + 2}\right) =$ **(4, –1)** **Ans.**

Now PQ = Distance between P (–2, 3) and Q (4, –1)

$= \sqrt{(4 + 2)^2 + (-1 - 3)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$

and, $BC = \sqrt{(12 + 6)^2 + (-3 - 9)^2} = \sqrt{324 + 144} = \sqrt{468} = 6\sqrt{13}$

$PQ = 2\sqrt{13}$ and $BC = 6\sqrt{13} \quad \Rightarrow \quad$ **$PQ = \dfrac{1}{3}BC$** **Ans.**

4 Calculate the ratio in which the line joining the points (4, 6) and (–5, –4) is divided by the line $y = 3$. Also, find the co-ordinates of the point of intersection.

Solution :

The co-ordinates of every point on the line $y = 3$ will be of the type $(x, 3)$.

Now, $y = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$ [Taking : $(x, 3) = (x, y)$, $(4, 6) = (x_1, y_1)$ and $(-5, -4) = (x_2, y_2)$]

$\Rightarrow \quad 3 = \dfrac{m_1 \times -4 + m_2 \times 6}{m_1 + m_2}$

$\Rightarrow \quad 3m_1 + 3m_2 = -4m_1 + 6m_2 \quad \Rightarrow \quad 7m_1 = 3m_2 \quad \Rightarrow \quad \dfrac{m_1}{m_2} = \dfrac{3}{7}$

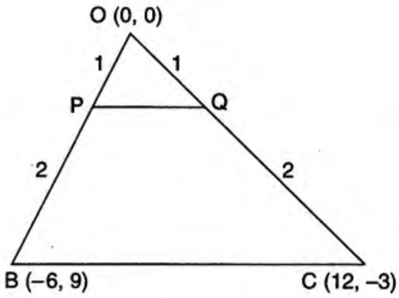
∴ **The required ratio is 3 : 7** **Ans.**

Now, $x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2} \quad \Rightarrow \quad x = \dfrac{3 \times -5 + 7 \times 4}{3 + 7} = \dfrac{13}{10}$

∴ **The required point of intersection =** $\left(\dfrac{13}{10}, 3\right)$ **Ans.**

5 The origin O, B (–6, 9) and C (12, –3) are vertices of triangle OBC. Point P divides OB in the ratio 1 : 2 and point Q divides OC in the ratio 1 : 2. Find the co-ordinates of points P and Q. Also, show that : $PQ = \dfrac{1}{3}BC$.

Solution :



For point P : $m_1 : m_2 = 1 : 2$, $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (-6, 9)$

$\therefore \; P = \left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$= \left(\dfrac{1 \times -6 + 2 \times 0}{1 + 2}, \dfrac{1 \times 9 + 2 \times 0}{1 + 2}\right)$

$=$ **(–2, 3)** **Ans.**

For point Q : $m_1 : m_2 = 1 : 2$, $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (12, -3)$

$\therefore \quad Q = \left(\dfrac{1 \times 12 + 2 \times 0}{1 + 2}, \dfrac{1 \times -3 + 2 \times 0}{1 + 2}\right) =$ **(4, –1)** **Ans.**

Now PQ = Distance between P (–2, 3) and Q (4, –1)

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and, $BC = \sqrt{(12 + 6)^2 + (-3 - 9)^2} = \sqrt{324 + 144} = \sqrt{468} = 6\sqrt{13}$

$PQ = 2\sqrt{13}$ and $BC = 6\sqrt{13} \quad \Rightarrow \quad$ $\mathbf{PQ = \dfrac{1}{3}BC}$ **Ans.**

13.3 Points of Trisection :

Let points P and Q lie on line segment AB and divide it into three equal parts *i.e.*, **AP = PQ = QB**; then P and Q are called **points of trisection** of AB.

6 Find the co-ordinates of the points of trisection of the line segment joining the points A (6, –2) and B (–8, 10).

Solution :

Let P and Q be the points of trisection so that AP = PQ = QB.

For P :

$m_1 : m_2 = AP : PB = 1 : 2; (x_1, y_1) = (6, -2)$ and $(x_2, y_2) = (-8, 10)$

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times -8 + 2 \times 6}{1 + 2} = \frac{4}{3}$$

$$\therefore \quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1 \times 10 + 2 \times -2}{1 + 2} = 2$$

$\therefore$ **Point P** $= \left(\frac{4}{3}, 2\right)$ **Ans.**

For Q :

$m_1 : m_2 = AQ : QB = 2 : 1; (x_1, y_1) = (6, -2)$ and $(x_2, y_2) = (-8, 10)$

$$\therefore Q = \left(\frac{2 \times -8 + 1 \times 6}{2 + 1}, \frac{2 \times 10 + 1 \times -2}{2 + 1}\right) = \left(-\frac{10}{3}, 6\right)$$ **Ans.**

7 Show that P (3, *m* –5) is a point of trisection of the line segment joining the points A (4, –2) and B (1, 4). Hence, find the value of '*m*'.

Solution :

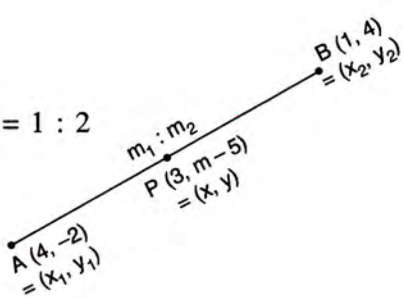
P will be a point of trisection of AB if it divides AB in the ratio 1 : 2 or 2 : 1.

Since, $x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$\Rightarrow \quad 3 = \frac{m_1 \times 1 + m_2 \times 4}{m_1 + m_2}$

$\Rightarrow \quad 3m_1 + 3m_2 = m_1 + 4m_2$

$\Rightarrow \quad 2m_1 = m_2$ and $\frac{m_1}{m_2} = \frac{1}{2}$ *i.e.* $m_1 : m_2 = 1 : 2$



Hence, P is a point of trisection of AB.

Now, $y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$\Rightarrow \quad m - 5 = \frac{1 \times 4 + 2 \times -2}{1 + 2}$

$\Rightarrow \quad$ $m = 5$ **Ans.**

EXERCISE 13(A)

1. Calculate the co-ordinates of the point P which divides the line segment joining :

   (i) A (1, 3) and B (5, 9) in the ratio 1 : 2

   (ii) A (–4, 6) and B (3, –5) in the ratio 3 : 2.

2. In what ratio is the line joining (2, –3) and (5, 6) divided by the x-axis ?

3. In what ratio is the line joining (2, –4) and (–3, 6) divided by the y-axis ?

4. In what ratio does the point (1, a) divide the join of (–1, 4) and (4, –1) ?

   Also, find the value of a.

5. In what ratio does the point (a, 6) divide the join of (–4, 3) and (2, 8) ?

   Also, find the value of a.

6. In what ratio is the join of (4, 3) and (2, –6) divided by the x-axis ? Also, find the co-ordinates of the point of intersection.

7. Find the ratio in which the join of (–4, 7) and (3, 0) is divided by the y-axis. Also, find the co-ordinates of the point of intersection.

8. Points A, B, C and D divide the line segment joining the point (5, –10) and the origin in five equal parts. Find the co-ordinates of B and D.

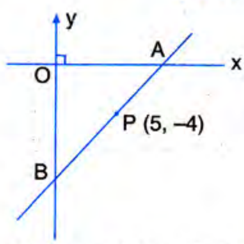
9. The line joining the points A (–3, –10) and B (–2, 6) is divided by the point P such that $\frac{PB}{AB} = \frac{1}{5}$. Find the co-ordinates of P.

10. P is a point on the line joining A (4, 3) and B (–2, 6) such that 5AP = 2BP. Find the co-ordinates of P.

11. Calculate the ratio in which the line joining the points (–3, –1) and (5, 7) is divided by the line $x = 2$. Also, find the co-ordinates of the point of intersection.

12. Calculate the ratio in which the line joining A (6, 5) and B (4, –3) is divided by the line $y = 2$. **[2006]**

13. The point P (5, –4) divides the line segment AB, as shown in the figure, in the ratio 2 : 5. Find the co-ordinates of points A and B.



14. Find the co-ordinates of the points of trisection of the line joining the points (–3, 0) and (6, 6).

15. Show that the line segment joining the points (–5, 8) and (10, –4) is trisected by the co-ordinate axes.

16. Show that A (3, –2) is a point of trisection of the line-segment joining the points (2, 1) and (5, –8).

    Also, find the co-ordinates of the other point of trisection.

17. If A = (–4, 3) and B = (8, –6)

    (i) Find the length of AB.

    (ii) In what ratio is the line joining A and B, divided by the x-axis ? **[2008]**

18. The line segment joining the points M (5, 7) and N (–3, 2) is intersected by the y-axis at point L. Write down the abscissa of L. Hence, find the ratio in which L divides MN.

    Also, find the co-ordinates of L.

19. A (2, 5), B (–1, 2) and C (5, 8) are the co-ordinates of the vertices of the triangle ABC. Points P and Q lie on AB and AC respectively, such that : AP : PB = AQ : QC = 1 : 2.

    (i) Calculate the co-ordinates of P and Q.

    (ii) Show that : PQ = $\frac{1}{3}$BC.

20. A (–3, 4), B (3, –1) and C (–2, 4) are the vertices of a triangle ABC. Find the length of line segment AP, where point P lies inside BC, such that BP : PC = 2 : 3.

21. The line segment joining A(2, 3) and B(6, –5) is intercepted by x-axis at the point K. Write down the ordinate of the point K. Hence, find the ratio in which K divides AB. Also, find the co-ordinates of the point K. **[2006]**

22. The line segment joining A(4, 7) and B(–6, –2) is intercepted by the y-axis at the point K. Write down the abscissa of the point K. Hence, find the ratio in which K divides AB. Also, find the co-ordinates of the point K.

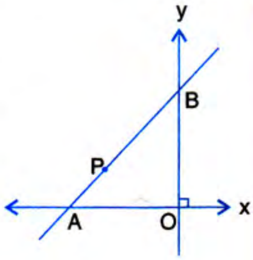
23. The line joining P(– 4, 5) and Q(3, 2) intersects the y-axis at point R. PM and QN are perpendiculars from P and Q on the x-axis. Find :

    (i) the ratio PR : RQ.

    (ii) the co-ordinates of R.

    (iii) the area of the quadrilateral PMNQ. **[2004]**

24. In the given figure, line APB meets the x-axis at point A and y-axis at point B. P is the point (– 4, 2) and AP : PB = 1 : 2. Find the co-ordinates of A and B. **[2013]**

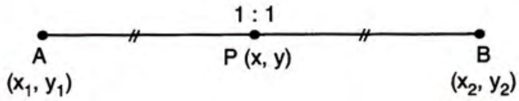


25. Given a line segment AB joining the points A(–4, 6) and B(8, –3). Find :
   (i) the ratio in which AB is divided by the y-axis.
   (ii) find the co-ordinates of the point of intersection.
   (iii) the length of AB. **[2012]**

13.4 Mid-Point Formula :

To find the co-ordinates of the mid-point of the line segment joining the two given fixed points.

Let P be the mid-point of the line segment joining the points A (x_1, y_1) and B (x_2, y_2).



Required to find the co-ordinates of P. Suppose P = (x, y).

For mid-point P, the ratio $m_1 : m_2 = 1 : 1$

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1\cdot x_2 + 1\cdot x_1}{1 + 1} = \frac{x_1 + x_2}{2}$$

$$\text{and, } y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1\cdot y_2 + 1\cdot y_1}{1 + 1} = \frac{y_1 + y_2}{2}$$

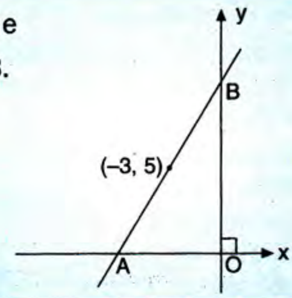
$\therefore$ Mid-point of the join of A (x_1, y_1) and B (x_2, y_2) = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

8 Find the co-ordinates of the mid-point of the line segment joining the points P (4, –6) and Q (–2, 4).

Solution :

$$\textbf{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 - 2}{2}, \frac{-6 + 4}{2}\right) = (1, -1) \qquad \textbf{Ans.}$$

9 The mid-point of line segment AB (shown in the diagram) is (–3, 5). Find the co-ordinates of A and B.



Solution :

Since, point A lies on the x-axis; let A = $(x, 0)$
Since, point B lies on the y-axis; let B = $(0, y)$

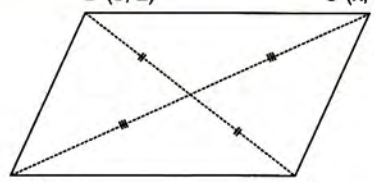
$$\text{Mid-point of AB} = \left(\frac{x + 0}{2}, \frac{0 + y}{2}\right) = (-3, 5)$$

$$\Rightarrow \quad \frac{x}{2} = -3; \ \frac{y}{2} = 5 \ \textit{i.e. } x = -6 \text{ and } y = 10$$

$\therefore$ Co-ordinates of **A = (–6, 0)** and co-ordinates of **B = (0, 10)** **Ans.**

10 A (14, –2), B (6, –2) and D (8, 2) are the three vertices of a parallelogram ABCD. Find the co-ordinates of the fourth vertex C.

Solution :



Let C = (x, y)

Since the diagonals of a parallelogram bisect each other;

$\therefore$ Mid-point of AC = mid-point of BD

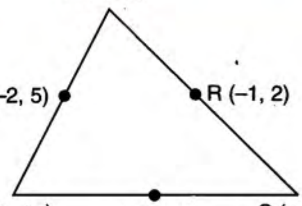
$$\Rightarrow \left(\frac{14+x}{2}, \frac{-2+y}{2}\right) = \left(\frac{8+6}{2}, \frac{2+-2}{2}\right)$$

$$\Rightarrow \frac{14+x}{2} = \frac{14}{2} \text{ and } \frac{-2+y}{2} = \frac{0}{2} \quad \Rightarrow \quad x = 0 \text{ and } y = 2$$

$\therefore$ **The vertex C = (0, 2)** **Ans.**

11 In triangle ABC, P (–2, 5) is mid-point of AB, Q (2, 4) is mid-point of BC and R (–1, 2) is mid-point of AC. Calculate the co-ordinates of vertices A, B and C.

Solution :



Let A = (x_1, y_1), B = (x_2, y_2) and C = (x_3, y_3).

Since, P is mid-point of AB

$$\Rightarrow \frac{x_1+x_2}{2} = -2 \text{ and } \frac{y_1+y_2}{2} = 5$$

i.e. $x_1 + x_2 = -4$ I

and, $y_1 + y_2 = 10$ II

Since, Q is mid-point of BC

$$\Rightarrow \frac{x_2+x_3}{2} = 2 \text{ and } \frac{y_2+y_3}{2} = 4$$

i.e. $x_2 + x_3 = 4$ III

and, $y_2 + y_3 = 8$ IV

Since, R is mid-point of AC

$$\Rightarrow \frac{x_1+x_3}{2} = -1 \text{ and } \frac{y_1+y_3}{2} = 2$$

i.e. $x_1 + x_3 = -2$ V

and, $y_1 + y_3 = 4$ VI

Adding equations I, III and V; we get :

$$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = -4 + 4 - 2$$

$$\Rightarrow 2(x_1 + x_2 + x_3) = -2$$

i.e. $x_1 + x_2 + x_3 = -1$ VII

On subtracting eq. I from eq. VII, we get : $x_3 = -1 + 4 = 3$

On subtracting eq. III from eq. VII, we get : $x_1 = -1 - 4 = -5$

And, on subtracting eq. V from eq. VII, we get : $x_2 = -1 + 2 = 1$

In the same way, on solving equations II, IV and VI, we get :

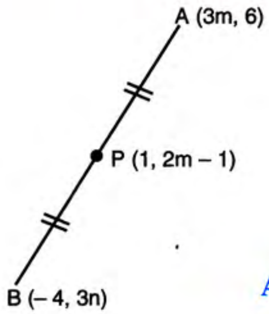
$$y_1 = 3,\ y_2 = 7 \text{ and } y_3 = 1$$

$\therefore$ **A = (x_1, y_1) = (–5, 3), B = (x_2, y_2) = (1, 7)** and **C = (x_3, y_3) = (3, 1)** **Ans.**

12 The mid-point of the line segment joining (3m, 6) and (– 4, 3*n*) is (1, 2*m* –1). Find the values of *m* and *n*. **[2006]**

Solution :

According to the adjoining figure, we have :



$$\frac{3m+(-4)}{2} = 1 \quad \text{and} \quad \frac{6+3n}{2} = 2m - 1$$

$\Rightarrow$ $3m - 4 = 2$ and $6 + 3n = 4m - 2$

$\Rightarrow$ $m = 2$ and $3n = 4m - 8$

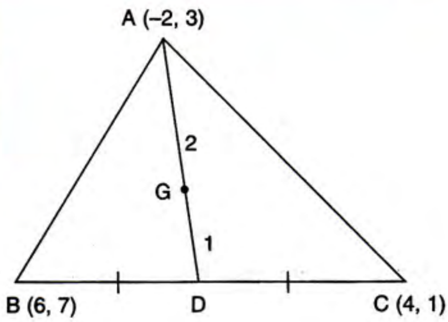
$\Rightarrow$ $3n = 4 \times 2 - 8$

$\Rightarrow$ $\mathbf{m = 2}$ and $\mathbf{n = 0}$ **Ans.**

13.5 Centroid of a triangle :

The **centroid** of a triangle is the point of intersection of its medians and it (centroid) divides each median in the ratio 2 : 1.

13 Find the co-ordinates of the point of intersection of the medians of triangle ABC; given A = (-2, 3), B = (6, 7) and C = (4, 1).

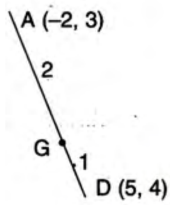


Solution :

Let D be the mid-point of BC.

$$\therefore D = \left(\frac{6+4}{2}, \frac{7+1}{2}\right) = (5, 4)$$

If G is the point of intersection of medians (centroid), it divides the median AD in the ratio 2 : 1.



$$\therefore G = \left[\frac{2 \times 5 + 1 \times -2}{2+1}, \frac{2 \times 4 + 1 \times 3}{2+1}\right]$$

$$= \left(\frac{8}{3}, \frac{11}{3}\right)$$ **Ans.**

Direct method : For the vertices A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) of triangle ABC, its centroid = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

Thus, in the case of example given above;

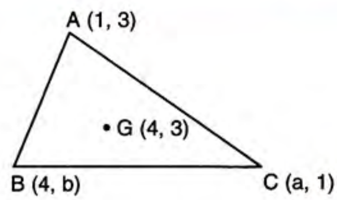
$$\text{Centroid} = \left(\frac{-2+6+4}{3}, \frac{3+7+1}{3}\right)$$

$$= \left(\frac{8}{3}, \frac{11}{3}\right)$$ **Ans.**

Taking :
$(-2, 3) = (x_1, y_1)$
$(6, 7) = (x_2, y_2)$
and $(4, 1) = (x_3, y_3)$

14 ABC is a triangle and G(4, 3) is the centroid of the triangle. If A = (1, 3), B = (4, *b*) and C = (*a*, 1), find '*a*' and '*b*'. Find the length of side BC. **[2011]**

Solution :



Since, G is centroid of Δ ABC.

$$\left(\frac{1+4+a}{3}, \frac{3+b+1}{3}\right) = (4, 3)$$

$\Rightarrow \quad \frac{5+a}{3} = 4 \quad$ and $\quad \frac{4+b}{3} = 3$

$\Rightarrow \quad a = 7 \quad$ and $\quad b = 5$ **Ans.**

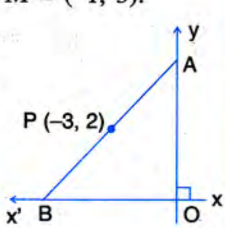
Clearly, B = (4, *b*) = (4, 5) and C = (*a*, 1) = (7, 1)

$\therefore$ BC $= \sqrt{(7-4)^2 + (1-5)^2} = \sqrt{9+16} = \sqrt{25} = 5$ units **Ans.**

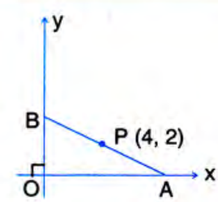
EXERCISE 13(B)

old - C

1. Find the mid-point of the line segment joining the points :
   (i) (–6, 7) and (3, 5) (ii) (5, –3) and (–1, 7)
2. Points A and B have co-ordinates (3, 5) and (*x*, *y*) respectively. The mid-point of AB is (2, 3). Find the values of *x* and *y*.
3. A (5, 3), B (–1, 1) and C (7, –3) are the vertices of triangle ABC. If L is the mid-point of AB and M is the mid-point of AC, show that : LM = $\frac{1}{2}$BC.
4. Given M is the mid-point of AB, find the co-ordinates of :
   (i) A; if M = (1, 7) and B = (–5, 10),
   (ii) B; if A = (3, –1) and M = (–1, 3).
5. P (–3, 2) is the mid-point of line segment AB as shown in the given figure. Find the co-ordinates of points A and B.



6. In the given figure, P (4, 2) is mid-point of line segment AB. Find the co-ordinates of A and B.



7. (–5, 2), (3, –6) and (7, 4) are the vertices of a triangle. Find the length of its median through the vertex (3, –6).
8. Given a line ABCD in which AB = BC = CD, B = (0, 3) and C = (1, 8).
   Find the co-ordinates of A and D.
9. One end of the diameter of a circle is (–2, 5). Find the co-ordinates of the other end of it, if the centre of the circle is (2, –1).
10. A (2, 5), B (1, 0), C (–4, 3) and D (–3, 8) are the vertices of quadrilateral ABCD. Find the co-ordinates of the mid-points of AC and BD. Give a special name to the quadrilateral.
11. P (4, 2) and Q (–1, 5) are the vertices of parallelogram PQRS and (–3, 2) are the co-ordinates of the point of intersection of its diagonals. Find co-ordinates of R and S.

12. A (–1, 0), B (1, 3) and D (3, 5) are the vertices of a parallelogram ABCD. Find the co-ordinates of vertex C.

13. The points (2, –1), (–1, 4) and (–2, 2) are mid-points of the sides of a triangle. Find its vertices.

14. Points A (–5, x), B (y, 7) and C (1, –3) are collinear (*i.e.* lie on the same straight line) such that AB = BC. Calculate the values of x and y.

15. Points P (a, –4), Q (–2, b) and R (0, 2) are collinear. If Q lies between P and R, such that PR = 2QR, calculate the values of a and b.

16. Calculate the co-ordinates of the centroid of the triangle ABC, if A = (7, –2), B = (0, 1) and C = (–1, 4).

17. The co-ordinates of the centroid of a triangle PQR are (2, –5). If Q = (–6, 5) and R = (11, 8); calculate the co-ordinates of vertex P.

18. A (5, x), B (–4, 3) and C (y, –2) are the vertices of the triangle ABC whose centroid is the origin. Calculate the values of x and y.

EXERCISE 13(C)

1. Given a triangle ABC in which A = (4, –4), B = (0, 5) and C = (5, 10). A point P lies on BC such that BP : PC = 3 : 2. Find the length of line segment AP.

2. A(20, 0) and B(10, –20) are two fixed points. Find the co-ordinates of the point P in AB such that : 3PB = AB. Also, find the co-ordinates of some other point Q in AB such that : AB = 6 AQ.

3. A(–8, 0), B(0, 16) and C(0, 0) are the vertices of a triangle ABC. Point P lies on AB and Q lies on AC such that AP : PB = 3 : 5 and AQ : QC = 3 : 5.

   Show that : PQ = $\frac{3}{8}$BC.

4. Find the co-ordinates of points of trisection of the line segment joining the point (6, –9) and the origin.

5. A line segment joining A(–1, $\frac{5}{3}$) and B(a, 5) is divided in the ratio 1 : 3 at P, the point where the line segment AB intersects the y-axis.

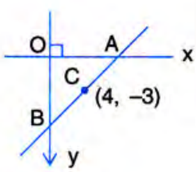
   (i) Calculate the value of 'a'.

   (ii) Calculate the co-ordinates of 'P'.

6. In what ratio is the line joining A(0, 3) and B(4, –1) divided by the x-axis ?

   Write the co-ordinates of the point where AB intersects the x-axis.

7. The mid-point of the segment AB, as shown in diagram, is C(4, –3). Write down the co-ordinates of A and B.



8. AB is a diameter of a circle with centre C = (–2, 5). If A = (3, –7), find

   (i) the length of radius AC.

   (ii) the coordinates of B. **[2013]**

9. Find the co-ordinates of the centroid of a triangle ABC whose vertices are :

   A(–1, 3), B(1, –1) and C(5, 1). **[2006]**

10. The mid-point of the line segment joining (4a, 2b – 3) and (– 4, 3b) is (2, –2a). Find the values of a and b.

11. The mid-point of the line segment joining (2a, 4) and (–2, 2b) is (1, 2a + 1). Find the values of a and b. **[2007]**

12. (i) Write down the co-ordinates of the point P that divides the line joining A(– 4, 1) and B(17, 10) in the ratio 1 : 2.

    (ii) Calculate the distance OP, where O is the origin.

    (iii) In what ratio does the y-axis divide the line AB ?

13. Prove that the points A(–5, 4); B(–1, –2) and C(5, 2) are the vertices of an isosceles right-angled triangle. Find the co-ordinates of D so that ABCD is a square.

14. M is the mid-point of the line segment joining the points A(–3, 7) and B(9, –1). Find the co-ordinates of point M. Further, if R(2, 2) divides the line segment joining M and the origin in the ratio p : q, find the ratio p : q.

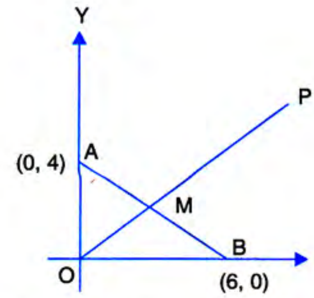
15. Calculate the ratio in which the line joining A(–4, 2) and B(3, 6) is divided by point P(x, 3). Also, find (i) x (ii) length of AP. **[2014]**

16. Find the ratio in which the line $2x + y = 4$ divides the line segment joining the points P(2, –2) and Q(3. 7).

17. If the abscissa of a point P is 2. Find the ratio in which this point divides the line segment joining the points (–4, 3) and (6, 3). Also, find the co-ordinates of point P.

18. The line joining the points (2, 1) and (5, –8) is trisected at the points P and Q. If point P lies on the line $2x - y + k = 0$, find the value of k. Also, find the co-ordinates of point Q.

19. M is the mid-point of the line segment joining the points A(0, 4) and B(6, 0). M also divides the line segment OP in the ratio 1 : 3. Find :
    (i) co-ordinates of M
    (ii) co-ordinates of P
    (iii) length of BP



20. Find the image of the point A(5, –3) under reflection in the point P(–1, 3).

21. A(–4, 2), B(0, 2) and C(–2, –4) are vertices of a triangle ABC. P, Q and R are mid-points of sides BC, CA and AB respectively. Show that the centroid of Δ PQR is the same as the centroid of Δ ABC.