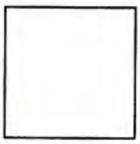
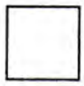
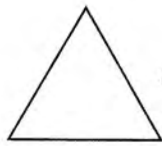
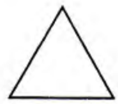
15 Similarity
(With Applications to Maps and Models)

15.1 Introduction :

1. **Similarity of Figures :** Two figures are said to be similar, if they have the *same shape* but may *differ in size.*

Examples :

(i)  and 

(ii)  and 

(a) *Same shape means :* Angles of one figure are equal to corresponding angles of other figure each to each.

(b) *Same size means :* Sides of one figure are equal to corresponding sides of the other figure each to each.

2. **Congruency of Figures :** Two figures are said to be *congruent,* if they have the *same shape* and the *same size.*

> *Congruent figures* are always *similar*, whereas *similar figures* are not necessarily *congruent.*

15.2 Similar Triangles :

Two triangles are said to be *similar*, if their corresponding angles are *equal* and corresponding sides are *proportional* (*i.e.* the ratios between the lengths of corresponding sides are equal).

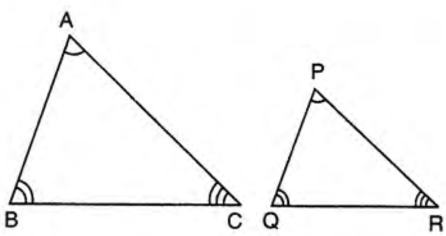
e.g. Triangles ABC and PQR are similar, if :

(a) their corresponding angles are equal

i.e. $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$

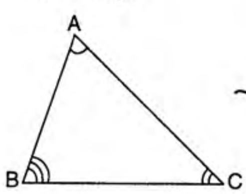
(b) their corresponding sides are in proportion

i.e. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$.



Symbolically, we write : Δ ABC ~ Δ PQR; where symbol ~ is read as, "is similar to".

In the same way;



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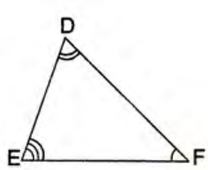
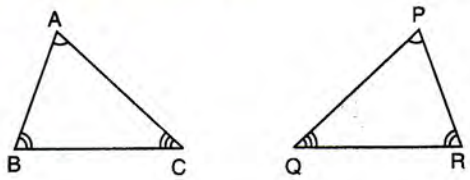


$\Leftrightarrow \begin{cases} \angle A = \angle F, \angle B = \angle E \text{ and } \angle C = \angle D. \\ \text{Also, } \frac{AB}{EF} = \frac{BC}{DE} = \frac{AC}{DF} \end{cases}$

15.3 Corresponding sides and corresponding angles :

1. In similar triangles, the *sides opposite to equal angles* are said to be the *corresponding sides.*

   e.g. the adjoining figure shows, $\Delta ABC \sim \Delta PRQ$ in which $\angle A = \angle P$, $\angle B = \angle R$ and $\angle C = \angle Q$.



$\therefore$ (i) $\angle A = \angle P$ $\Rightarrow$ Sides opposite to $\angle A$ and $\angle P$ are the corresponding sides.

$\Rightarrow$ Sides BC and QR are the corresponding sides.

(ii) $\angle B = \angle R$ $\Rightarrow$ Sides opposite to $\angle B$ and $\angle R$ are the corresponding sides

$\Rightarrow$ Sides AC and PQ are the corresponding sides.

Similarly, (iii) $\angle C = \angle Q$ $\Rightarrow$ Sides AB and PR are the corresponding sides.

As, the triangles ABC and PRQ are similar; we have :

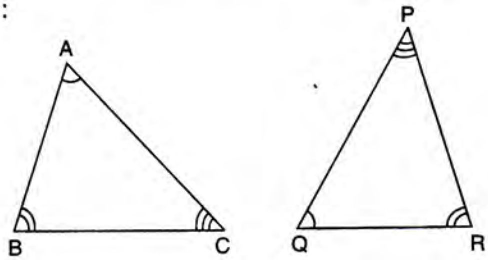
$\frac{BC}{QR} = \frac{AC}{PQ} = \frac{AB}{PR}$ [Corresponding sides of similar triangles are proportional]

In the same way, if in Δ ABC and Δ PQR :

$\angle A = \angle Q$, $\angle B = \angle R$ and $\angle C = \angle P$;

the two triangles are similar

i.e. Δ ABC is similar to Δ QRP



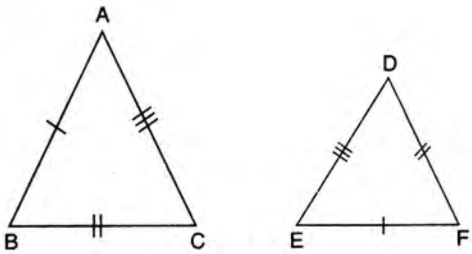
And, $\frac{\text{side opp. to } \angle A}{\text{side opp. to } \angle Q} = \frac{\text{side opp. to } \angle B}{\text{side opp. to } \angle R}$

$= \frac{\text{side opp. to } \angle C}{\text{side opp. to } \angle P} \Rightarrow \frac{BC}{PR} = \frac{AC}{PQ} = \frac{AB}{QR}$

2. In similar triangles, *the angles opposite to proportional sides* are the corresponding angles and so, they *are equal.*

   e.g. in similar triangles ABC and EFD,

   if $\frac{AB}{EF} = \frac{BC}{DF} = \frac{AC}{DE}$, then



(i) ratio $\frac{AB}{EF}$ $\Rightarrow$ angle opposite to side AB = angle opposite to side EF

i.e. $\angle C = \angle D$,

(ii) ratio $\frac{BC}{DF}$ $\Rightarrow$ angle opposite to side BC = angle opposite to side DF

i.e. $\angle A = \angle E$ and

(iii) ratio $\frac{AC}{DE}$ $\Rightarrow$ angle opposite to side AC = angle opposite to side DE

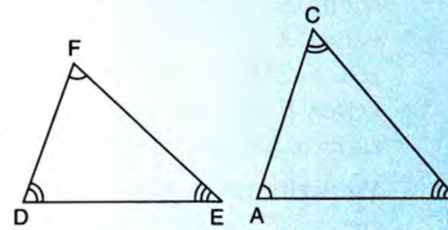
i.e. $\angle B = \angle F$.

1. Please note that *in congruent triangles* the *corresponding sides* are equal, whereas *in similar triangles,* the *corresponding sides* are in *proportion*.

   e.g. in the adjoining figures; if Δ ABC and Δ FED are congruent, then BC = DE, AB = EF and AC = DF.

   But, if Δ ABC and Δ FED are similar, then

   $$\frac{BC}{DE} = \frac{AB}{EF} = \frac{AC}{DF}$$



2. Triangles, which are similar to the same triangle, are similar to each other also.

15.4 Condition of Similar Triangle : (SAS, AA or AAA and SSS)

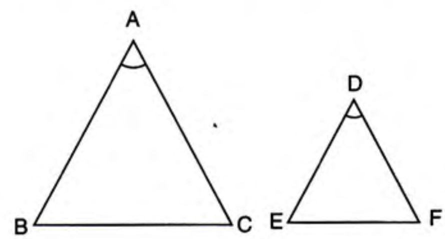
1. *If one angle of a triangle is equal to any angle of the other triangle and in both the triangles, the sides including the equal angles are in proportion; then the triangles are similar.* (**SAS** postulate)

Example :

If in Δ ABC and Δ DEF,

$\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$

then by SAS, Δ ABC ~ Δ DEF.



Similarly, if $\angle B = \angle E$ and $\frac{AB}{DE} = \frac{BC}{EF}$, then also Δ ABC ~ Δ DEF and so on.

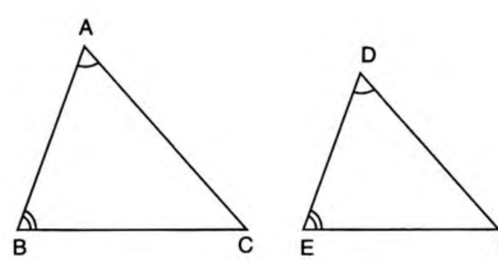
2. *If two triangles have atleast two pairs of corresponding angles equal; the triangles are similar.* (**AA** or **AAA** postulate)

Example :

If in Δ ABC and Δ DEF,

$\angle A = \angle D$ and $\angle B = \angle E$, Δ ABC ~ Δ DEF.

> Since, the sum of the angles of a triangle is 180°, therefore if two angles of one triangle are equal to two angles of another triangle each to each, their third angles are also equal.



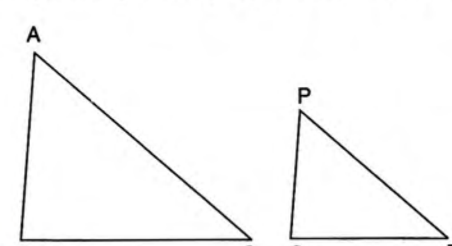
3. *If two triangles have their three pairs of corresponding sides proportional, then the triangles are similar.* (**SSS** postulate)

Example :

If in Δ ABC and Δ PQR,

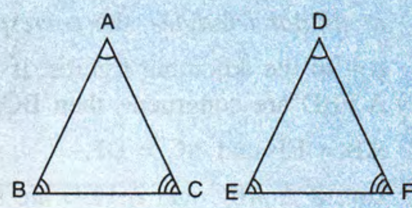
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

Δ ABC ~ Δ PQR.



Important :

1. The given figure shows two similar triangles ABC and DEF such that vertex A corresponds to vertex D (as, ∠A = ∠D); vertex B corresponds to vertex E (as, ∠B = ∠E) and, vertex C corresponds to vertex F (as, ∠C = ∠F).



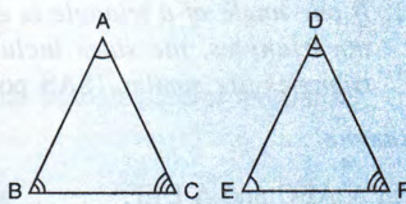
   We write : Δ ABC ~ Δ DEF and not Δ ABC ~ Δ DFE or Δ BAC ~ Δ DEF, etc.

   In fact, the order of vertices of two similar triangles must be written in such a way that the corresponding vertices occupy the same position.

   Thus, triangle ABC is similar to triangle DEF

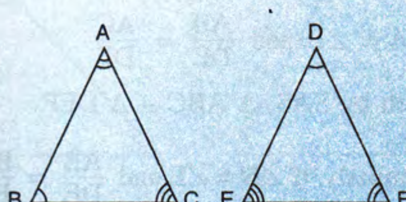
   ⇒ Δ ABC ~ Δ DEF [A ↔ D, B ↔ E and C ↔ F]

2. The adjoining figure shows two similar triangles such that, the corresponding vertices are:
   A ↔ E; B ↔ D and C ↔ F.



   ∴ Δ ABC ~ Δ EDF

3. In the adjoining figure, the corresponding vertices are : A and F, B and D and C and E.

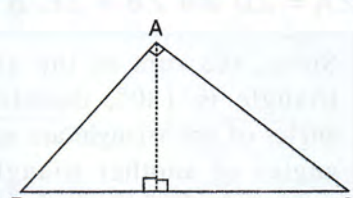


   ∴ Δ ABC ~ Δ FDE

1 A perpendicular drawn from the vertex of the right angle of a right-angled triangle divides the triangle into two triangles similar to each other and also to the original triangle. Prove it.

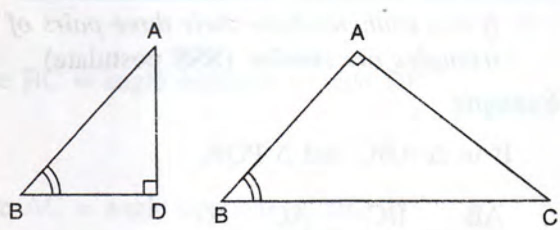
Solution :

Let ABC be a right angled triangle, right angled at vertex A and AD is perpendicular drawn to the hypotenuse BC.



To Prove : Δ DBA ~ Δ DAC ~ Δ ABC

Proof :



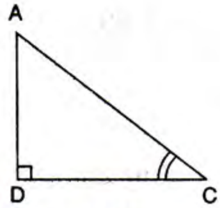
Statement :	**Reason :**
1. In Δ DBA and Δ ABC :	
(i) ∠ABD = ∠ABC	[Common]
(ii) ∠ADB = ∠BAC	[Each 90°]
∴ Δ DBA ~ Δ ABC	[AA or AAA postulate]

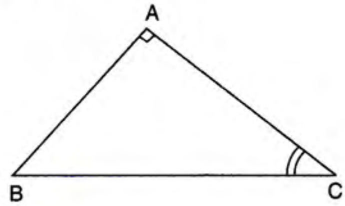
2. In Δ DAC and Δ ABC :

(i) $\angle ACD = \angle ACB$ [Common]

(ii) $\angle ADC = \angle BAC$ [Each 90°]

$\therefore$ Δ DAC ~ Δ ABC [AA or AAA postulate]



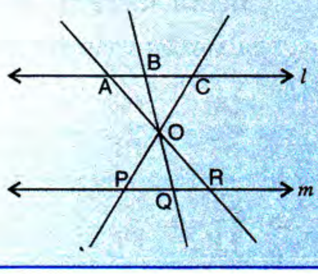


3. $\therefore$ **Δ DBA ~ Δ DAC ~ Δ ABC** [From 1 and 2]

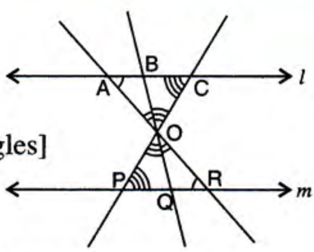
Hence Proved.

2 In the given figure, lines *l* and *m* are parallel. Three concurrent lines through point O meet line *l* at points A, B and C; and line *m* at points P, Q and R as shown.

Prove that : $\frac{AB}{BC} = \frac{QR}{PQ}$.



Solution :



Since, line *l* is parallel to line *m* and AR is transversal

$\therefore$ $\angle OAB = \angle ORQ$ [Alternate angles]

Also, $\angle AOB = \angle QOR$ [Vertically opposite angles]

$\therefore$ $\Delta OAB \sim \Delta ORQ$ [By A.A. postulate]

$\Rightarrow$ $\frac{AB}{QR} = \frac{OB}{OQ}$ ...... I [Corresponding sides of similar triangles are in proportion]

Similarly, $\angle OCB = \angle OPQ$ [Alternate angles]

and, $\angle BOC = \angle POQ$ [Vertically opposite angles]

$\therefore$ $\Delta\ BOC \sim \Delta\ QOP$ [By A.A. postulate]

$\Rightarrow$ $\frac{BC}{PQ} = \frac{OB}{OQ}$ ...... II [Corresponding sides of similar Δs]

From equations I and II, we get :

$$\frac{AB}{QR} = \frac{BC}{PQ}$$

$$\Rightarrow \quad \frac{AB}{BC} = \frac{QR}{PQ}$$

Hence proved.

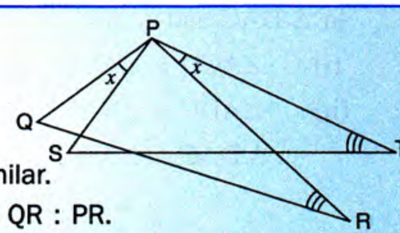
3 In the figure, given alongside,

$\angle QPS = \angle RPT$

and $\angle PRQ = \angle PTS$.

(i) Prove that triangles PQR and PST are similar.

(ii) If PT : ST = 3 : 4; find the ratio between QR : PR.



Solution :

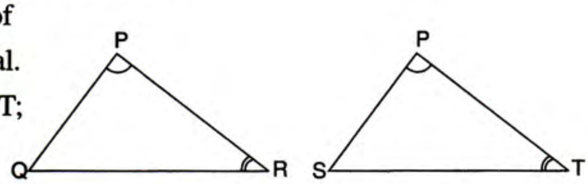
(i) In Δ PQR and Δ PST,

1. $\angle QPR = \angle SPT$ [Each equal to $\angle x + \angle SPR$]
2. $\angle PRQ = \angle PTS$ [Given]

$\therefore$ **Δ PQR ~ Δ PST** [AA postulate]

(ii) Since the corresponding sides of similar triangles are proportional.

$\therefore$ In similar triangles PQR and PST;

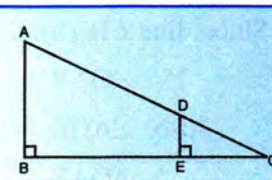


$$\frac{QR}{ST} = \frac{PR}{PT}$$

$\Rightarrow \dfrac{QR}{PR} = \dfrac{ST}{PT} = \dfrac{4}{3}$ $\left[\text{Given, } \dfrac{PT}{ST} = \dfrac{3}{4} \Rightarrow \dfrac{ST}{PT} = \dfrac{4}{3}\right]$

$\Rightarrow$ **QR : PR = 4 : 3** **Ans.**

4 In the given figure, AB and DE are perpendiculars to BC. If AB = 9 cm, DE = 3 cm and AC = 24 cm, calculate AD. **[2005]**



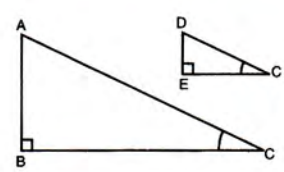
Solution :

In Δ ABC and Δ DEC :

(i) $\angle ABC = \angle DEC$ [Each 90°]

(ii) $\angle$ C is common

$\therefore$ Δ ABC ≅ Δ DEC [By A.A.]



$\Rightarrow \dfrac{AC}{DC} = \dfrac{AB}{DE}$ *i.e.* $\dfrac{24\text{ cm}}{DC} = \dfrac{9\text{ cm}}{3\text{ cm}}$

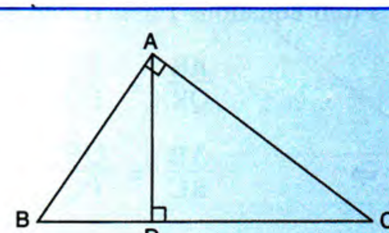
$\Rightarrow DC = \dfrac{24 \times 3}{9}$ cm = 8 cm

$\therefore$ **AD** = AC − DC = 24 cm − 8 cm = **16 cm** **Ans.**

5 In the adjoining figure, ABC is a triangle right-angled at vertex A and AD is altitude.

(i) Prove that : Δ ABD is similar to Δ CAD.

(ii) If BD = 3·6 cm and CD = 6·4 cm; find the length of AD.



Solution :

(i) In Δ CAD, let $\angle C = x$

$\Rightarrow \quad \angle CAD = 90° - x$

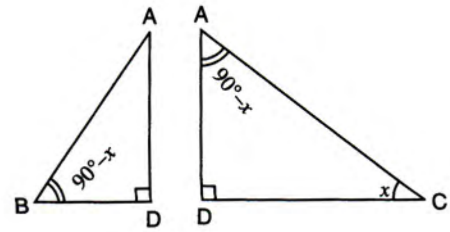
In Δ ABC,

$\angle C = x \Rightarrow \angle B = 90° - x$

Now in Δ ABD and Δ CAD,

1. $\angle ADB = \angle CDA$ [Each is 90°]
2. $\angle ABD = \angle CAD$ [Each is $90° - x$]

$\therefore$ **Δ ABD ~ Δ CAD** [By AA] **Hence Proved.**

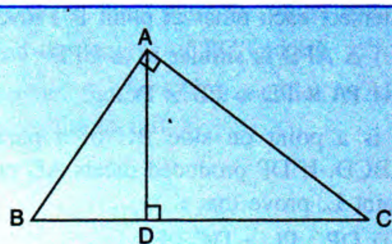


(ii) Since Δ ABD ~ Δ CAD $\Rightarrow \dfrac{AD}{CD} = \dfrac{BD}{AD}$

i.e. $AD^2 = BD \times CD = 3{\cdot}6 \times 6{\cdot}4$

$\Rightarrow$ **AD = 4·8 cm** **Ans.**

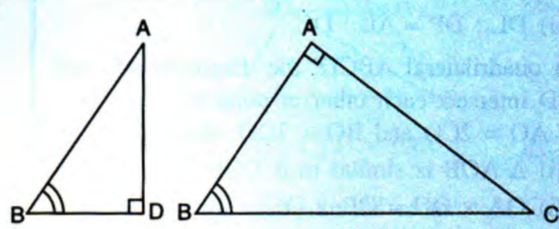
In Δ ABC, with $\angle A = 90°$ and $AD \perp BC$, we get three pairs of similar triangles.



1. **Δ ABD ~ Δ CBA** [By AA]

$\Rightarrow \dfrac{AB}{BC} = \dfrac{BD}{AB}$

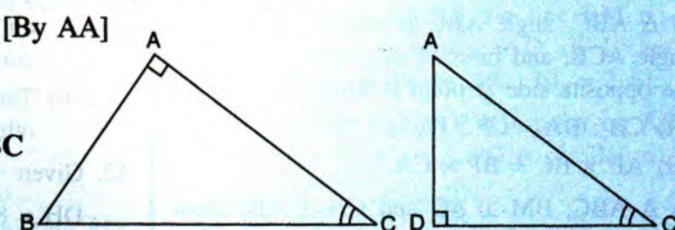
$\Rightarrow$ $\mathbf{AB^2 = BD \times BC}$



2. **Δ ABC ~ Δ DAC** [By AA]

$\Rightarrow \dfrac{AC}{DC} = \dfrac{BC}{AC}$

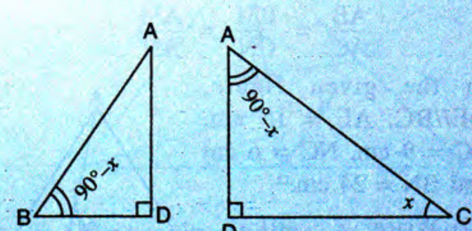
$\Rightarrow$ $\mathbf{AC^2 = DC \times BC}$



3. **Δ BAD ~ Δ ACD** [By AA]

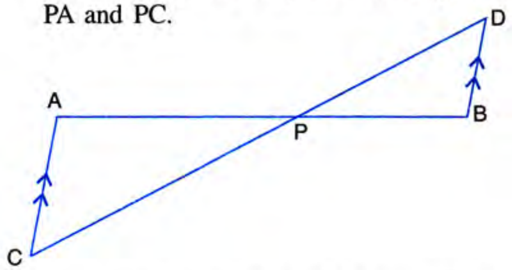
$\Rightarrow \dfrac{AD}{DC} = \dfrac{BD}{AD}$

$\Rightarrow$ $\mathbf{AD^2 = BD \times DC}$



EXERCISE 15(A)

1. In the figure, given below, straight lines AB and CD intersect at P; and AC // BD. Prove that :
   (i) Δ APC and Δ BPD are similar.
   (ii) If BD = 2·4 cm, AC = 3·6 cm, PD = 4·0 cm and PB = 3·2 cm; find the lengths of PA and PC.



2. In a trapezium ABCD, side AB is parallel to side DC; and the diagonals AC and BD intersect each other at point P. Prove that :
   (i) Δ APB is similar to Δ CPD.
   (ii) PA × PD = PB × PC.

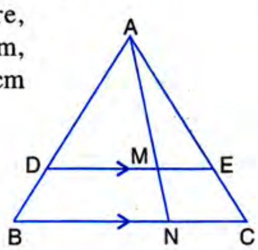
3. P is a point on side BC of a parallelogram ABCD. If DP produced meets AB produced at point L, prove that :
   (i) DP : PL = DC : BL.
   (ii) DL : DP = AL : DC.

4. In quadrilateral ABCD, the diagonals AC and BD intersect each other at point O.
   If AO = 2CO and BO = 2DO; show that :
   (i) Δ AOB is similar to Δ COD.
   (ii) OA × OD = OB × OC.

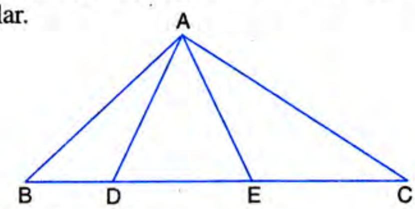
5. In Δ ABC, angle ABC is equal to twice the angle ACB, and bisector of angle ABC meets the opposite side at point P. Show that :
   (i) CB : BA = CP : PA
   (ii) AB × BC = BP × CA

6. In Δ ABC; BM ⊥ AC and CN ⊥ AB; show that :
$$\frac{AB}{AC} = \frac{BM}{CN} = \frac{AM}{AN}$$

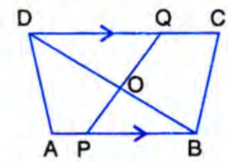
7. In the given figure, DE//BC, AE = 15 cm, EC = 9 cm, NC = 6 cm and BN = 24 cm.
   (i) Write all possible pairs of similar triangles.
   (ii) Find lengths of ME and DM.



8. In the given figure, AD = AE and $AD^2 = BD \times EC$.
   Prove that : triangles ABD and CAE are similar.



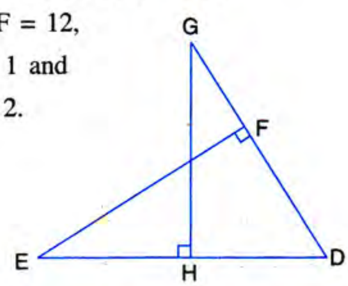
9. In the given figure, AB // DC, BO = 6 cm and DQ = 8 cm; find: BP × DO.



10. Angle BAC of triangle ABC is obtuse and AB = AC. P is a point in BC such that PC = 12 cm. PQ and PR are perpendiculars to sides AB and AC respectively. If PQ = 15 cm and PR = 9 cm; find the length of PB.

11. State, true or false :
    (i) Two similar polygons are necessarily congruent.
    (ii) Two congruent polygons are necessarily similar.
    (iii) All equiangular triangles are similar.
    (iv) All isosceles triangles are similar.
    (v) Two isosceles-right triangles are similar.
    (vi) Two isosceles triangles are similar, if an angle of one is congruent to the corresponding angle of the other.
    (vii) The diagonals of a trapezium divide each other into proportional segments.

12. Given : ∠GHE = ∠DFE = 90°,
    DH = 8, DF = 12,
    DG = 3x – 1 and
    DE = 4x + 2.

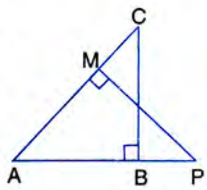


    Find : the lengths of segments DG and DE.

13. D is a point on the side BC of triangle ABC such that angle ADC is equal to angle BAC. Prove that : $CA^2 = CB \times CD$.

14. In the given figure, Δ ABC and Δ AMP are right angled at B and M respectively.

Given AC = 10 cm, AP = 15 cm and PM = 12 cm.



(i) Prove that : Δ ABC ~ Δ AMP

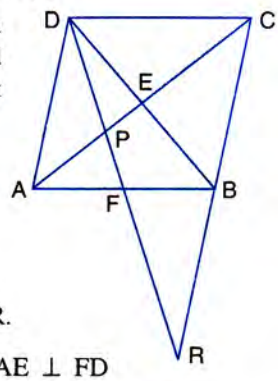
(ii) Find : AB and BC. **[2012]**

15. Given : RS and PT are altitudes of Δ PQR. Prove that :

(i) Δ PQT ~ Δ QRS,

(ii) PQ × QS = RQ × QT.

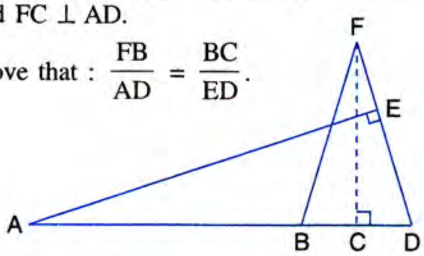
16. Given : ABCD is a rhombus, DPR and CBR are straight lines.



Prove that :
DP × CR = DC × PR.

17. Given : FB = FD, AE ⊥ FD and FC ⊥ AD.

Prove that : $\frac{FB}{AD} = \frac{BC}{ED}$.



18. In Δ PQR, $\angle$Q = 90° and QM is perpendicular to PR. Prove that :

(i) $PQ^2 = PM \times PR$

(ii) $QR^2 = PR \times MR$

(iii) $PQ^2 + QR^2 = PR^2$

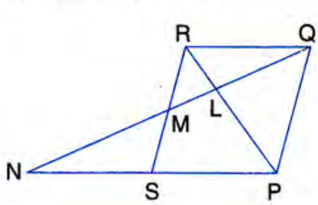
19. In Δ ABC, $\angle$B = 90° and BD ⊥ AC.

(i) If CD = 10 cm and BD = 8 cm; find AD.

(ii) If AC = 18 cm and AD = 6 cm; find BD.

(iii) If AC = 9 cm and AB = 7 cm; find AD.

20. In the figure, PQRS is a parallelogram with PQ = 16 cm and QR = 10 cm. L is a point on PR such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N.



Find the lengths of PN and RM.

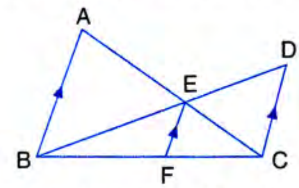
21. In quadrilateral ABCD, diagonals AC and BD intersect at point E such that

AE : EC = BE : ED.

Show that : ABCD is a trapezium.

22. In triangle ABC, AD is perpendicular to side BC and $AD^2 = BD \times DC$.
Show that angle BAC = 90°.

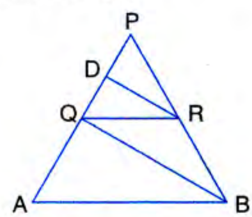
23. In the given figure, AB // EF // DC; AB = 67·5 cm, DC = 40·5 cm and AE = 52·5 cm.



(i) Name the three pairs of similar triangles.

(ii) Find the lengths of EC and EF.

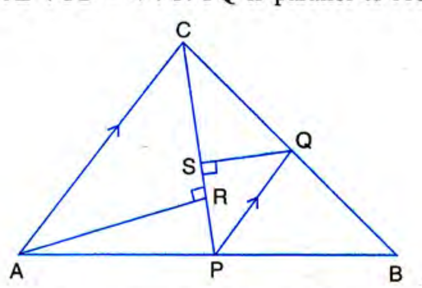
24. In the given figure, QR is parallel to AB and DR is parallel to QB.



Prove that : $PQ^2 = PD \times PA$.

25. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting diagonal AC in L and AD produced in E. Prove that : EL = 2 BL.

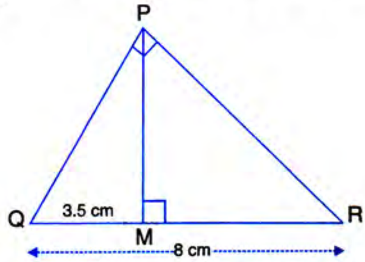
26. In the given figure, P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC.



(i) Calculate the ratio PQ : AC, giving reason for your answer.

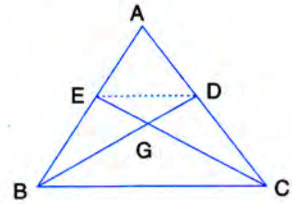
(ii) In triangle ARC, $\angle$ARC = 90° and in triangle PQS, $\angle$PSQ = 90°. Given QS = 6 cm, calculate the length of AR.

27. In the right-angled triangle QPR, PM is an altitude.



Given that QR = 8 cm and MQ = 3·5 cm, calculate the value of PR. **[2000]**

28. In the figure, given below, the medians BD and CE of a triangle ABC meet at G. Prove that :



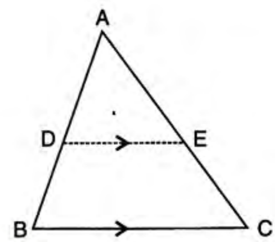
(i) Δ EGD ~ Δ CGB and

(ii) BG = 2 GD from (i) above.

15.5 Basic Proportionality Theorem With Applications :

Theorem 1

A line drawn parallel to one side of a triangle divides the other two sides proportionally.



Given : In Δ ABC; line DE is drawn parallel to side BC which meets AB at D and AC at E.

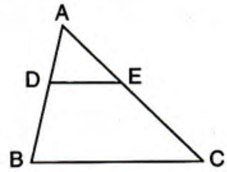
To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Proof :

Statement :	Reason :
In Δ ABC and Δ ADE,	
1. $\angle ABC = \angle ADE$	[Corresponding angles]
2. $\angle ACB = \angle AED$	[Corresponding angles]
3. $\angle BAC = \angle DAE$	[Common]
$\therefore \Delta ABC \sim \Delta ADE$	[AAA postulate]
$\therefore \frac{AB}{AD} = \frac{AC}{AE}$	[Corresponding sides of similar triangles are proportional]
$\Rightarrow \frac{AD+DB}{AD} = \frac{AE+EC}{AE}$	
$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	$[\frac{AD}{AD} = 1$ and $\frac{AE}{AE} = 1]$
$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$	[Cancelling 1 from both the sides]
$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$	[Taking the reciprocal]

Hence Proved.

Conversely : *If a line divides any two sides of a triangle proportionally, the line is parallel to the third side.*

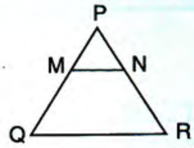


Note : In Δ ABC, D is a point in AB and E is a point AC such that $\frac{AD}{BD} = \frac{AE}{CE}$, then DE//BC.

1. M and N are points on sides PQ and PR respectively of Δ PQR; then :

   (i) if MN//QR $\Rightarrow \frac{PM}{MQ} = \frac{PN}{NR}$

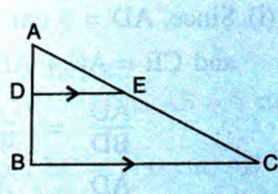
   and, (ii) if $\frac{PM}{MQ} = \frac{PN}{NR} \Rightarrow$ MN//QR



2. In Δ ABC, DE is parallel to BC

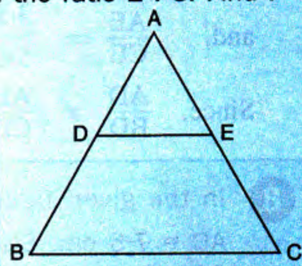
   $\Rightarrow$ (i) $\frac{AD}{BD} = \frac{AE}{CE}$

   and, (ii) $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ (As, Δ ADE ~ Δ ABC)



6 In the adjoining figure; DE // BC and D divides AB in the ratio 2 : 3. Find :

(i) $\frac{AE}{EC}$ (ii) $\frac{AE}{AC}$ (iii) DE, if BC = 7·5 cm.



Solution :

(i) Since, a line drawn parallel to one side of a triangle, divides the other two sides proportionally;

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ I

Also, given that D divides AB in the ratio 2 : 3 *i.e.* $\frac{AD}{DB} = \frac{2}{3}$ II

From I and II, we get : $\mathbf{\frac{AE}{EC} = \frac{2}{3}}$ **Ans.**

(ii) $\frac{AE}{AC} = \frac{AE}{AE + EC} = \frac{2}{2+3} = \frac{2}{5}$ **Ans.**

(iii) $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ [As, Δ ADE ~ Δ ABC]

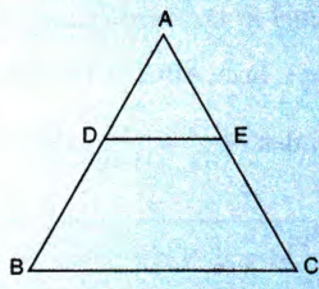
$\Rightarrow \quad \frac{DE}{7{\cdot}5} = \frac{2}{5}$ [From (ii), $\frac{AE}{AC} = \frac{2}{5}$]

$\Rightarrow \quad$ **DE** $= \frac{2}{5} \times 7{\cdot}5$ cm = **3 cm** **Ans.**

7 In Δ ABC, D and E are points on the sides AB and AC respectively. Find whether DE // BC; if :

(i) AD = 3 cm, BD = 4·5 cm, AE = 4 cm and AC = 10 cm

(ii) AB = 7 cm, BD = 4·5 cm, AE = 3·5 cm and CE = 5·6 cm



Solution :

DE will be parallel to BC, only when $\frac{AD}{BD} = \frac{AE}{CE}$

(i) Since, AD = 3 cm, BD = 4·5 cm; AE = 4 cm and CE = AC − AE = 10 cm − 4 cm = 6 cm.

$\therefore \quad \frac{AD}{BD} = \frac{3}{4 \cdot 5} = \frac{2}{3}$ and, $\frac{AE}{CE} = \frac{4}{6} = \frac{2}{3}$

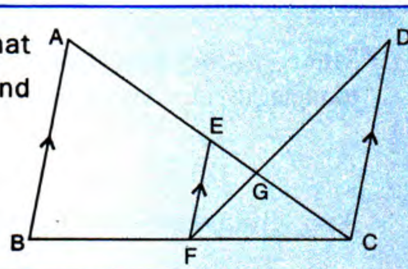
$\Rightarrow \quad \frac{AD}{BD} = \frac{AE}{CE} \qquad \therefore$ **DE // BC**

(ii) $\frac{AD}{BD} = \frac{AB - BD}{BD} = \frac{7 - 4 \cdot 5}{4 \cdot 5} = \frac{2 \cdot 5}{4 \cdot 5} = \frac{5}{9}$ **Ans.**

and, $\frac{AE}{CE} = \frac{3 \cdot 5}{5 \cdot 6} = \frac{5}{8}$

Since, $\frac{AD}{BD} \neq \frac{AE}{CE} \Rightarrow$ **DE is not parallel to BC.** **Ans.**

8 In the given figure; AB//EF//CD. Given that AB = 7·5 cm, EG = 2·5 cm, GC = 5 cm and DC = 9 cm. Calculate : (i) EF (ii) AC.



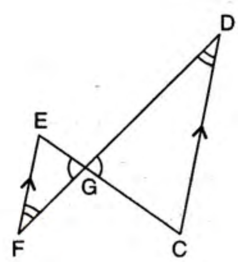
Solution :

(i) In Δ EGF and Δ CGD,

$\angle EGF = \angle CGD$ [Vertically opposite angles]

and, $\angle EFG = \angle D$ [Alternate angles]

$\therefore \quad \Delta EGF \sim \Delta CGD$ [By A.A. postulate]



$\Rightarrow \quad \frac{EF}{DC} = \frac{EG}{GC}$ [Corresponding parts of similar triangles are proportional]

$\Rightarrow \quad \frac{EF}{9\text{ cm}} = \frac{2 \cdot 5\text{ cm}}{5\text{ cm}}$

$\Rightarrow \quad$ **EF** $= \frac{2 \cdot 5}{5} \times 9$ cm = **4.5 cm** **Ans.**

(ii) $EC = EG + GC$

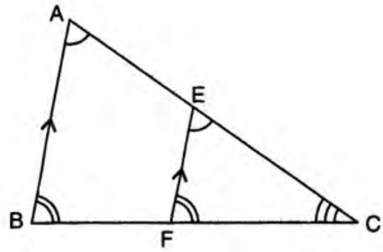
$= 2{\cdot}5 \text{ cm} + 5 \text{ cm} = 7{\cdot}5 \text{ cm}$

$\Delta ABC \sim \Delta EFC$ [By A.A.A.]

$\Rightarrow \frac{AC}{EC} = \frac{AB}{EF}$

$\Rightarrow \frac{AC}{7{\cdot}5 \text{ cm}} = \frac{7{\cdot}5 \text{ cm}}{4{\cdot}5 \text{ cm}}$

$\Rightarrow$ **AC** $= \frac{7{\cdot}5 \times 7{\cdot}5}{4{\cdot}5}$ cm = **12·5 cm** **Ans.**



EXERCISE 15(B)

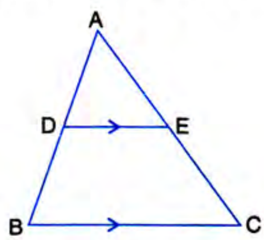
1. In the following figure, point D divides AB in the ratio 3 : 5. Find :

(i) $\frac{AE}{EC}$ (ii) $\frac{AD}{AB}$ (iii) $\frac{AE}{AC}$

Also, if:

(iv) DE = 2·4 cm, find the length of BC.

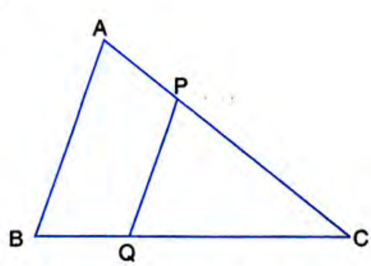
(v) BC = 4·8 cm, find the length of DE.



2. In the given figure, PQ // AB; CQ = 4·8 cm QB = 3·6 cm and AB = 6·3 cm. Find :

(i) $\frac{CP}{PA}$ (ii) PQ

(iii) If AP = x, then the value of AC in terms of x.



3. A line PQ is drawn parallel to the side BC of Δ ABC which cuts side AB at P and side AC at Q. If AB = 9·0 cm, CA = 6·0 cm and AQ = 4·2 cm, find the length of AP.

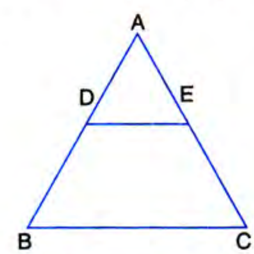
4. In Δ ABC, D and E are the points on sides AB and AC respectively.

Find whether DE // BC, if :

(i) AB = 9 cm, AD = 4 cm, AE = 6 cm and EC = 7·5 cm.

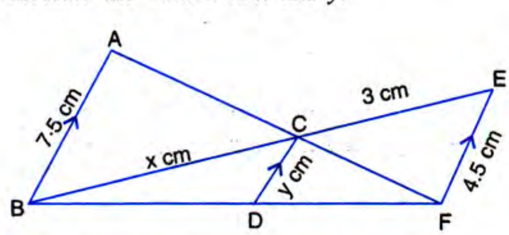
(ii) AB = 6·3 cm, EC = 11·0 cm, AD = 0·8 cm and AE = 1·6 cm.

5. In the given figure, Δ ABC ~ Δ ADE. If AE : EC = 4 : 7 and DE = 6·6 cm, find BC. If 'x' be the length of the perpendicular from A to DE, find the length of perpendicular from A to BC in terms of 'x'.



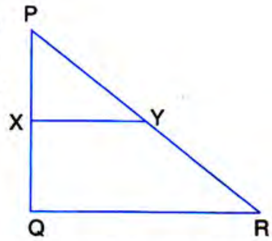
6. A line segment DE is drawn parallel to base BC of Δ ABC which cuts AB at point D and AC at point E. If AB = 5 BD and EC = 3·2 cm, find the length of AE.

7. In the figure, given below, AB, CD and EF are parallel lines. Given AB = 7·5 cm, DC = y cm, EF = 4·5 cm, BC = x cm and CE = 3 cm, calculate the values of x and y.

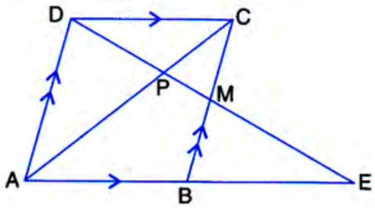


8. In the figure, given below, PQR is a right-angle triangle right angled at Q. XY is parallel

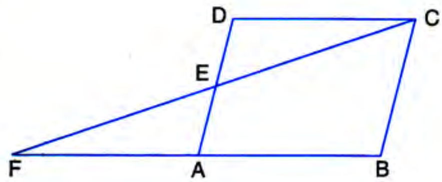
to QR, PQ = 6 cm, PY = 4 cm and PX : XQ = 1 : 2. Calculate the lengths of PR and QR.



9. In the following figure, M is mid-point of BC of a parallelogram ABCD. DM intersects the diagonal AC at P and AB produced at E. Prove that : PE = 2 PD



10. The given figure shows a parallelogram ABCD. E is a point in AD and CE produced meets BA produced at point F. If AE = 4 cm, AF = 8 cm and AB = 12 cm, find the perimeter of the parallelogram ABCD.



15.6 Relation between the areas of two triangles :

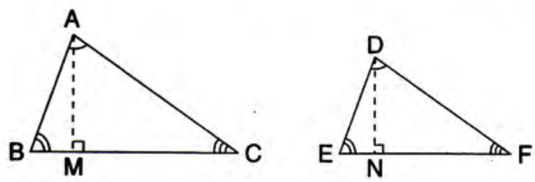
Theorem 2

The areas of two similar triangles are proportional to the squares on their corresponding sides.

Given : Δ ABC ~ Δ DEF

such that $\angle$BAC = $\angle$EDF,

$\angle$B = $\angle$E and $\angle$C = $\angle$F.



To Prove :

$$\frac{\text{Area of } \Delta\text{ ABC}}{\text{Area of } \Delta\text{ DEF}} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Construction :

Draw AM $\perp$ BC and DN $\perp$ EF.

Proof :

	Statement	*Reason*
1.	Area of Δ ABC $= \frac{1}{2}$BC $\times$ AM	Area of Δ $= \frac{1}{2}$base $\times$ altitude
	Area of Δ DEF $= \frac{1}{2}$EF $\times$ DN	Area of Δ $= \frac{1}{2}$base $\times$ altitude
	$\Rightarrow \frac{\text{Area of } \Delta\text{ ABC}}{\text{Area of } \Delta\text{ DEF}} = \frac{\frac{1}{2}BC \times AM}{\frac{1}{2}EF \times DN}$	
	$= \frac{BC}{EF} \times \frac{AM}{DN}$	I

2. In Δ ABM and Δ DEN :

(i) $\angle B = \angle E$ [Given]

(ii) $\angle AMB = \angle DNE$ [Each angle being 90°]

$\therefore$ Δ ABM ~ Δ DEN [By AA postulate]

$$\Rightarrow \frac{AM}{DN} = \frac{AB}{DE} \quad .....II$$

[Corresponding sides of similar triangles are in proportion]

3. Since, Δ ABC ~ Δ DEF [Given]

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad .....III$$

[Corresponding sides of similar triangles are in proportion]

$$\therefore \frac{AM}{DN} = \frac{BC}{EF}$$ [From II and III]

Substituting $\frac{AM}{DN} = \frac{BC}{EF}$ in equation I, we get :

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2} \quad .....IV$$

Now combining eq. III and eq. IV, we get :

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$ **Hence Proved.**

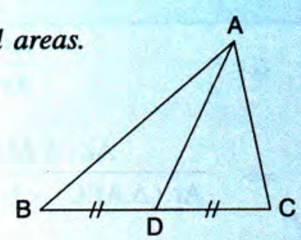
Remember :

1. *Median divides the triangle into two triangles of equal areas.*

   In the given figure, AD is median

   $\Rightarrow$ Area of Δ ABD = Area of Δ ACD

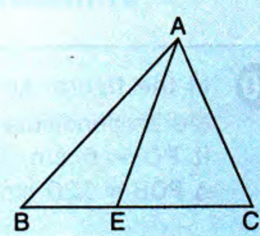
   $= \frac{1}{2} \times$ Area of Δ ABC.



2. *If two or many triangles have the common vertex and their bases are along the same straight line, the ratio between their areas is equal to the ratio between the lengths of their bases.*

   In the given figure, triangles ABE and ACE have the common vertex at point A and their bases are along the same straight line BC.

   $$\Rightarrow \frac{\text{Area of } \Delta ABE}{\text{Area of } \Delta ACE} = \frac{BE}{CE}.$$

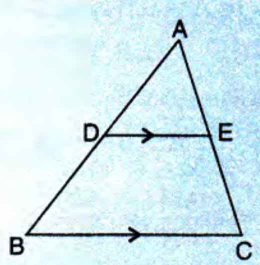


9 In the given figure, DE // BC.

(i) Prove that Δ ADE and Δ ABC are similar.

(ii) Given that AD = $\frac{1}{2}$ BD, calculate DE, if BC = 4·5 cm. [2004]

Also, find $\frac{\text{Ar. }(\Delta\text{ ADE})}{\text{Ar. }(\Delta\text{ ABC})}$ and $\frac{\text{Ar. }(\Delta\text{ ADE})}{\text{Ar. (trapezium BCED)}}$



Solution :

(i) DE // BC

$\Rightarrow$ $\angle ADE = \angle ABC$ [Corresponding angles]

and, $\angle AED = \angle ACB$ [Corresponding angles]

$\therefore$ **Δ ADE ~ Δ ABC** [By A.A.]

Hence Proved.

(ii) Since the corresponding sides of the similar triangles are in proportion :

$$\therefore \quad \frac{DE}{BC} = \frac{AD}{AB}$$

i.e. $$\frac{DE}{4{\cdot}5\text{ cm}} = \frac{1}{3}$$

$$\Rightarrow \quad \mathbf{DE} = \frac{1}{3} \times 4{\cdot}5 \text{ cm} = \mathbf{1{\cdot}5 \text{ cm}} \quad \textbf{Ans.}$$

Given, $AD = \frac{1}{2} BD$

Since, $\frac{AD}{BD} = \frac{1}{2}$

$$\Rightarrow \quad \frac{AD}{AD+BD} = \frac{1}{1+2}$$

$$\Rightarrow \quad \frac{AD}{AB} = \frac{1}{3}$$

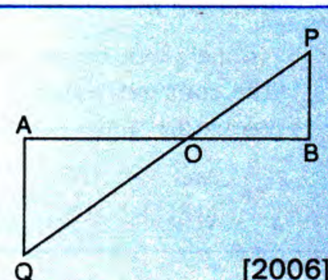
We know the ratio between the areas of two similar triangles = ratio between the squares of its corresponding sides.

$$\therefore \quad \frac{\text{Ar. }(\Delta\text{ ADE})}{\text{Ar. }(\Delta\text{ ABC})} = \frac{DE^2}{BC^2} = \frac{(1{\cdot}5)^2}{(4{\cdot}5)^2} = \frac{1}{9} \quad \textbf{Ans.}$$

$$\Rightarrow \quad \frac{\text{Ar. }(\Delta\text{ ADE})}{\text{Ar. }(\Delta\text{ ABC}) - \text{Ar. }(\Delta\text{ ADE})} = \frac{1}{9-1}$$

$$\Rightarrow \quad \frac{\text{Ar. }(\Delta\text{ ADE})}{\text{Ar. (trapezium BCED)}} = \frac{1}{8} \quad \textbf{Ans.}$$

10 In the figure, given alongside, PB and QA are perpendiculars to the line segment AB. If PO = 6 cm, QO = 9 cm and area of Δ POB = 120 cm^2, find the area of Δ QOA.



[2006]

Solution :

In Δ POB and Δ QOA :

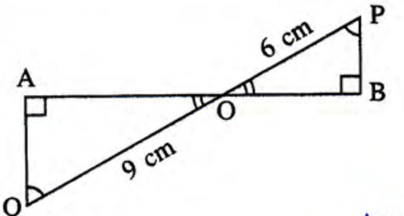
1. $\angle PBO = \angle QAO$ [Each 90°]

2. $\angle POB = \angle QOA$ [Vertically opposite angles]

$\Rightarrow \Delta POB \sim \Delta QOA$ [By A.A.]

$\Rightarrow \frac{Ar.(\Delta POB)}{Ar.(\Delta QOA)} = \frac{PO^2}{QO^2}$ [Ratio between the areas of two similar triangles is equal to the ratio between the squares of the corresponding sides]

$\Rightarrow \frac{120}{Ar.(\Delta QOA)} = \frac{6^2}{9^2}$



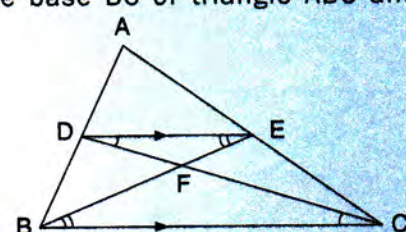
$\Rightarrow$ **Ar. (ΔQOA)** $= \frac{120\times 81}{36} cm^2$

$= \mathbf{270\ cm^2}$ **Ans.**

11 In the given figure, DE is parallel to the base BC of triangle ABC and AD : DB = 5 : 3. Find the ratio :

(i) $\frac{AD}{AB}$ and then $\frac{DE}{BC}$.

(ii) $\frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta DEC}$.



Solution :

(i) $\frac{AD}{DB} = \frac{5}{3} \Rightarrow \frac{AD}{AD + DB} = \frac{5}{5+3}$

$\Rightarrow \mathbf{\frac{AD}{AB} = \frac{5}{8}}$ **Ans.**

Since, Δ ADE ~ Δ ABC [By AA postulate]

$\Rightarrow \mathbf{\frac{DE}{BC}} = \frac{AD}{AB} = \mathbf{\frac{5}{8}}$ **Ans.**

(ii) Since, Δ DEF and Δ DEC have common vertex at E and their bases DF and DC are along the same straight line

$\therefore \frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta DEC} = \frac{DF}{DC}$ .....(I)

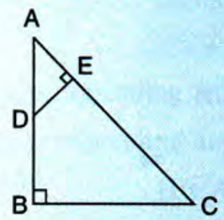
Now, show that Δ DFE ~ Δ CFB

$$\Rightarrow \quad \frac{DF}{FC} = \frac{DE}{BC} = \frac{5}{8}$$

$$\Rightarrow \quad \frac{DF}{DF + FC} = \frac{5}{5+8} \quad \Rightarrow \quad \frac{DF}{DC} = \frac{5}{13} \qquad \text{.....(II)}$$

From I and II, we get : $\frac{\text{Area of } \Delta \text{ DEF}}{\text{Area of } \Delta \text{ DEC}} = \frac{5}{13}$ **Ans.**

12 In Δ ABC, ∠B = 90°, AB = 12 cm and AC = 15 cm. D and E are points on AB and AC respectively such that ∠AED = 90° and DE = 3 cm. Calculate the area of Δ ABC and then the area of Δ ADE.



Solution :

In right-angled triangle ABC,

$$AB^2 + BC^2 = AC^2 \qquad \text{[Pythagoras theorem]}$$

$$\Rightarrow \quad BC^2 = 15^2 - 12^2$$

$$= 225 - 144 = 81$$

$$BC = 9 \text{ cm.}$$

$$\therefore \quad \text{Area of } \Delta \text{ ABC} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times BC \times AB$$

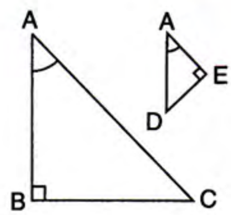
$$= \frac{1}{2} \times 9 \times 12 \text{ cm}^2 = 54 \text{ cm}^2 \qquad \textbf{Ans.}$$

In Δ ADE and Δ ABC,

$$\angle ABC = \angle AED = 90°$$

and ∠A is common to both.

∴ By A.A. axiom, Δ ADE ~ Δ ABC



$$\Rightarrow \quad \frac{\text{Ar. of } \Delta \text{ ADE}}{\text{Ar. of } \Delta \text{ ABC}} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \quad \frac{\text{Ar. of } \Delta \text{ ADE}}{54 \text{ cm}^2} = \frac{3^2}{9^2}$$

$$\Rightarrow \quad \text{Ar. of } \Delta \text{ ADE} = \frac{9}{81} \times 54 \text{ cm}^2$$

$$= 6 \text{ cm}^2 \qquad \textbf{Ans.}$$

1. (i) The ratio between the corresponding sides of two similar triangles is 2 is to 5. Find the ratio between the areas of these triangles.

   (ii) Areas of two similar triangles are 98 sq. cm and 128 sq. cm. Find the ratio between the lengths of their corresponding sides.

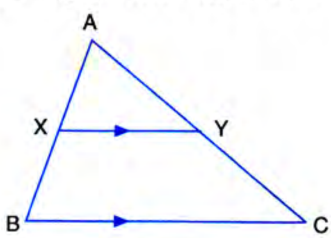
2. A line PQ is drawn parallel to the base BC of Δ ABC which meets sides AB and AC at points P and Q respectively. If AP = $\frac{1}{3}$PB; find the value of :

   (i) $\frac{\text{Area of } \Delta\text{ ABC}}{\text{Area of } \Delta\text{ APQ}}$

   (ii) $\frac{\text{Area of } \Delta\text{ APQ}}{\text{Area of trapezium PBCQ}}$

3. The perimeters of two similar triangles are 30 cm and 24 cm. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

4. In the given figure, AX : XB = 3 : 5



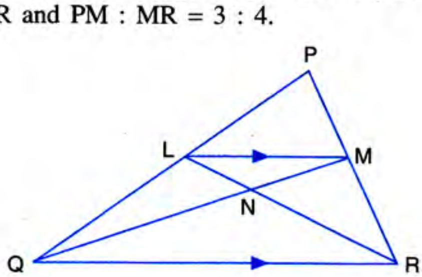
   Find :

   (i) the length of BC, if the length of XY is 18 cm.

   (ii) the ratio between the areas of trapezium XBCY and triangle ABC.

5. ABC is a triangle. PQ is a line segment intersecting AB in P and AC in Q such that PQ // BC and divides triangle ABC into two parts equal in area. Find the value of ratio BP : AB.

6. In the given triangle PQR, LM is parallel to QR and PM : MR = 3 : 4.



   Calculate the value of ratio :

   (i) $\frac{PL}{PQ}$ and then $\frac{LM}{QR}$

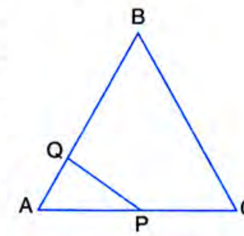
   (ii) $\frac{\text{Area of } \Delta\text{ LMN}}{\text{Area of } \Delta\text{ MNR}}$ (iii) $\frac{\text{Area of } \Delta\text{ LQM}}{\text{Area of } \Delta\text{ LQN}}$

7. The given diagram shows two isosceles triangles which are similar. In the given diagram, PQ and BC are not parallel; PC = 4, AQ = 3, QB = 12, BC = 15 and AP = PQ.

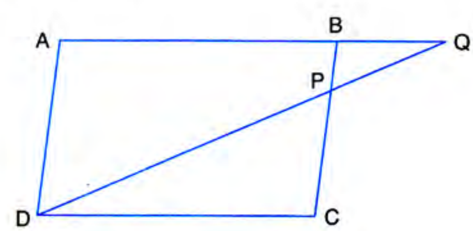
   Calculate :

   (i) the length of AP,

   (ii) the ratio of the areas of triangle APQ and triangle ABC.



8. In the figure, given below, ABCD is a parallelogram. P is a point on BC such that BP : PC = 1 : 2. DP produced meets AB produced at Q. Given the area of triangle CPQ = 20 cm^2.



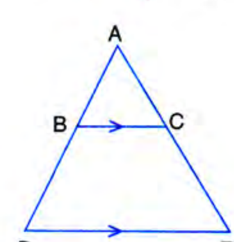
   Calculate :

   (i) area of triangle CDP,

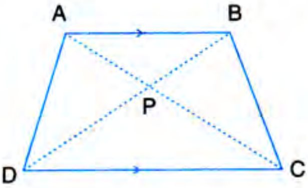
   (ii) area of parallelogram ABCD.

9. In the given figure, BC is parallel to DE. Area of triangle ABC = 25 cm^2, Area of trapezium BCED = 24 cm^2 and DE = 14 cm. Calculate the length of BC.

   Also, find the area of triangle BCD.



10. The given figure shows a trapezium in which AB is parallel to DC and diagonals AC and BD intersect at point P. If AP : CP = 3 : 5,



Find :

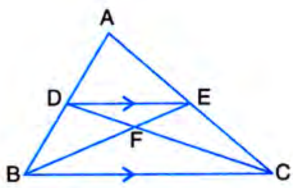
(i) Δ APB : Δ CPB (ii) Δ DPC : Δ APB

(iii) Δ ADP : Δ APB (iv) Δ APB : Δ ADB

11. In the given figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$.

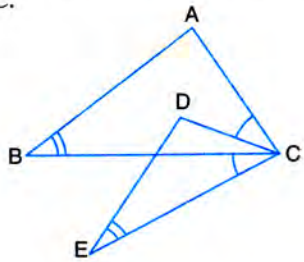
(i) Determine the ratios $\frac{AD}{AB}$ and $\frac{DE}{BC}$.

(ii) Prove that Δ DEF is similar to Δ CBF. Hence, find $\frac{EF}{FB}$.



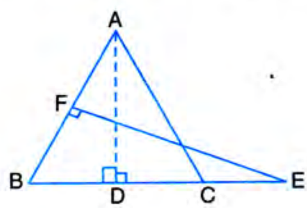
(iii) What is the ratio of the areas of Δ DEF and Δ BFC ? **[2007]**

12. In the given figure, ∠B = ∠E, ∠ACD = ∠BCE, AB = 10·4 cm and DE = 7·8 cm. Find the ratio between areas of the Δ ABC and Δ DEC.



13. Triangle ABC is an isosceles triangle in which AB = AC = 13 cm and BC = 10 cm. AD is perpendicular to BC. If CE = 8 cm and EF ⊥ AB, find :

(i) $\frac{\text{area of } \Delta \text{ ADC}}{\text{area of } \Delta \text{ FEB}}$ (ii) $\frac{\text{area of } \Delta \text{ FEB}}{\text{area of } \Delta \text{ ABC}}$

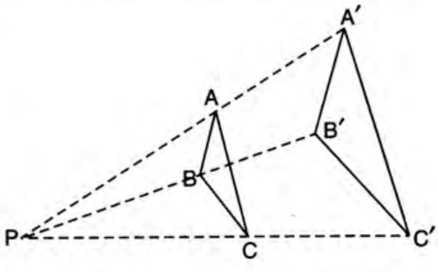


15.7 Similarity as a Size Transformation :

Draw a triangle ABC and mark a point P outside it.

Join P and A. Produce PA to point A′ such that PA′ = 2PA. Similarly, get PB′ = 2PB and PC′ = 2PC.

On joining A′, B′ and C′; we get another triangle A′ B′ C′ which is similar to the original triangle ABC. If the sides of the triangle A′ B′ C′ be measured and compared with the corresponding sides of the original triangle ABC, we find :

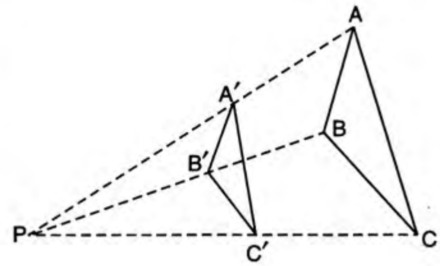


A′ B′ = 2AB, B′C′ = 2BC and C′A′ = 2CA.

Since, each side of the resulting triangle A′ B′ C′ is **twice** the corresponding side of the original triangle ABC; ΔA′B′C′ is similar to ABC and we say, that the triangle ABC has been **enlarged** by a **scale factor** 2 about the **centre of enlargement** P.

Similarly, if for the same triangle ABC and a point P outside it, we take a point A′ in PA such that PA′ = $\frac{1}{2}$PA; a point B′ in PB such that PB′ = $\frac{1}{2}$ PB and a point C′ in

PC such that $PC' = \frac{1}{2}PC$. On joining A′, B′ and C′ we again get a triangle A′ B′ C′ which is similar to the original triangle ABC.



On comparing the resulting triangle A′ B′ C′ and the original triangle ABC, it will be observed that $A'B' = \frac{1}{2}AB$, $B'C' = \frac{1}{2}BC$ and $C'A' = \frac{1}{2}CA$.

Since, each side of the resulting triangle A′ B′ C′ is **half** of the corresponding side of the given triangle ABC; we say, that the triangle ABC has been **reduced** by a **scale factor** $\frac{1}{2}$ about the **centre of reduction** P.

The type of **enlargement** or **reduction**, as discussed above, is called **size transformation.**

Thus in a size transformation, a given figure is enlarged or reduced by a scale factor k, such that the resulting figure (the image) is similar to the given figure (the object or the pre-image).

In a size transformation :

(i) the image of a line is a line,

(ii) the image of a triangle is a triangle,

(iii) the image of a quadrilateral is a quadrilateral and so on.

15.8 Applications to Maps and Models :

Maps and Models : Students use different maps of India and of Asia in Geography. Consider the map of India in which the positions of its major cities are shown. Measure the distance between any two cities marked in the map and compare it with the actual distance between those two cities to get the ratio between the two distances. In the same way, choose two more cities marked in the map and compare their distance (on the map) with the actual distance between them to get the ratio between the two distances; we find that the ratio of distances in both the cases is the same.

In fact, this ratio is already marked (written) on every map and is known as ***scale factor*** which is denoted by letter, ***'k'***.

For Example :

If the scale of a map is 1 : 20,000, this implies that, a distance of one cm on the map is equal to an actual distance of 20,000 cm (0·2 km) on the ground. And, also scale factor $k = \frac{1}{20{,}000}$.

The same principle is applicable to models. In the case of models,

$$\frac{\textit{Height of the model}}{\textit{Height of the object}} = \frac{\textit{Length of the model}}{\textit{Length of the object}}$$

$$= \frac{\textit{Width of the model}}{\textit{Width of the object}}$$

$$= \text{Scale factor } (k)$$

If the scale factor is k, then :

(i) each side of the resulting figure (the image) is k *times* the corresponding side of the given figure (the object or the pre-image).

(ii) the area of the resulting figure is k^2 *times* the area of the given figure.

(iii) in the case of solids, the volume of the resulting figure is k^3 *times* the volume of the given figure.

(iv) $k > 1 \Rightarrow$ the transformation is an enlargement,

$k < 1 \Rightarrow$ the transformation is a reduction and

$k = 1 \Rightarrow$ the transformation is an identity transformation.

13 A model of a ship is made to a scale of 1 : 200. If the length of the model is 4 *m*; calculate the length of the ship.

Solution :

Clearly, the scale factor $k = \frac{1}{200}$

And, the length of model = k ***times*** the length of the ship.

$\Rightarrow$ $4\,m = \frac{1}{200} \times$ the length of the ship

$\Rightarrow$ **The length of the ship = 800 m** **Ans.**

14 The scale of map is 1 : 50,000. In the map, a triangular plot ABC of land has the following dimensions :

AB = 2 cm, BC = 3·5 cm and angle ABC = 90°.

Calculate : (i) the actual length of side BC, in km, of the land.

(ii) the area of the plot in sq. km.

Solution :

Clearly, scale factor $k = \frac{1}{50,000}$

(i) Length of side BC in the map = k *times* the actual length of side BC in the land.

$\Rightarrow$ $3\cdot5 \text{ cm} = \frac{1}{50,000} \times$ actual length of BC

$\Rightarrow$ Actual length of BC $= 50{,}000 \times 3{\cdot}5$ cm

$= \mathbf{1{\cdot}75\ km}$ **Ans.**

(ii) Since, the area of triangle ABC in the map $= \frac{1}{2} \times 2 \text{ cm} \times 3{\cdot}5 \text{ cm}$

$= 3{\cdot}5$ sq. cm

And, the area of Δ ABC in the map $= k^2$ *times* the actual area of triangular plot ABC.

$$\Rightarrow \quad 3{\cdot}5 \text{ cm}^2 = \left(\frac{1}{50{,}000}\right)^2 \times \text{actual area of the plot}$$

$\Rightarrow$ **Actual area of the plot** $= 3{\cdot}5 \times 50{,}000 \times 50{,}000$ sq. cm

$= \mathbf{0{\cdot}875\ sq.\ km}$ **Ans.**

15 A rectangular tank has length = 4 m, width = 3 m and capacity = 30 m^3. A small model of the tank is made with capacity 240 cm^3. Find :

(i) the dimensions of the model.

(ii) the ratio between the total surface area of the tank and its model.

Solution :

For the tank :

Its length × breadth × height = Volume

$$\Rightarrow \quad 4 \text{ m} \times 3 \text{ m} \times \text{height} = 30 \text{ m}^3$$

$$\Rightarrow \quad \text{Height} = \frac{30}{4 \times 3} \text{ m} = 2{\cdot}5 \text{ m.}$$

Let the scale factor for reduction $= k$

$$\therefore \quad \text{Volume of the model} = k^3 \times \text{volume of the tank}$$

$$\Rightarrow \quad 240 \text{ cm}^3 = k^3 \times 30 \text{ m}^3$$

$$\Rightarrow \quad 240 \text{ cm}^3 = k^3 \times 30 \times 100 \times 100 \times 100 \text{ cm}^3$$

$$\Rightarrow \quad k^3 = \frac{240}{30 \times 100 \times 100 \times 100}$$

$$= \frac{1}{125000}$$

$$\Rightarrow \quad \text{Scale factor, } k = \frac{1}{50}$$

(i) $\therefore$ Length of the model $= k \times$ length of the tank

$$= \frac{1}{50} \times 4 \text{ m} = 8 \text{ cm,}$$

breadth of the model $= k \times$ breadth of the tank

$$= \frac{1}{50} \times 3 \text{ m} = 6 \text{ cm}$$

and, height of the model $= \frac{1}{50} \times 2{\cdot}5 \text{ m} = 5 \text{ cm}$

$\therefore$ **Dimensions of the model = 8 cm × 6 cm × 5 cm** **Ans.**

(ii) Since, total surface area of the model $= k^2 \times$ total S.A. of the tank

$$\Rightarrow \frac{\text{Total S.A. of the tank}}{\text{Total S.A. of the model}} = \frac{1}{k^2} = (50)^2$$

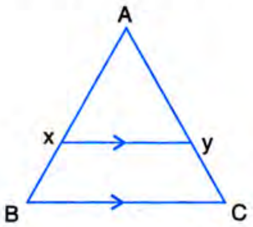
$= 2500 =$ **2500 : 1** **Ans.**

EXERCISE 15(D)

1. A triangle ABC has been enlarged by scale factor m = 2·5 to the triangle A′ B′ C′ Calculate :
   (i) the length of AB, if A′ B′ = 6 cm.
   (ii) the length of C′ A′ if CA = 4 cm.
2. A triangle LMN has been reduced by scale factor 0·8 to the triangle L′ M′ N′. Calculate:
   (i) the length of M′ N′, if MN = 8 cm.
   (ii) the length of LM, if L′ M′ = 5·4 cm.
3. A triangle ABC is enlarged, about the point O as centre of enlargement, and the scale factor is 3. Find :
   (i) A′ B′, if AB = 4 cm.
   (ii) BC, if B′ C′ = 15 cm.
   (iii) OA, if OA′ = 6 cm.
   (iv) OC′, if OC = 21 cm.
   Also, state the value of :
   (a) $\frac{OB'}{OB}$ (b) $\frac{C'A'}{CA}$
4. A model of an aeroplane is made to a scale of 1 : 400. Calculate :
   (i) the length, in cm, of the model; if the length of the aeroplane is 40 m.
   (ii) the length, in m, of the aeroplane, if length of its model is 16 cm.
5. The dimensions of the model of a multistorey building are 1·2 m × 75 cm × 2 m. If the scale factor is 1 : 30; find the actual dimensions of the building.
6. On a map drawn to a scale of 1 : 2,50,000; a triangular plot of land has the following measurements : AB = 3 cm, BC = 4 cm and angle ABC = 90°.
   Calculate :
   (i) the actual lengths of AB and BC in km.
   (ii) the area of the plot in sq. km.
7. A model of a ship is made to a scale of 1 : 200.
   (i) The length of the model is 4 m; calculate the length of the ship.
   (ii) The area of the deck of the ship is 160000 m^2; find the area of the deck of the model.
   (iii) The volume of the model is 200 litres; calculate the volume of the ship in m^3.
8. An aeroplane is 30 m long and its model is 15 cm long. If the total outer surface area of the model is 150 cm^2, find the cost of painting the outer surface of the aeroplane at the rate of ₹ 120 per sq. m. Given that 50 sq. m of the surface of the aeroplane is left for windows.

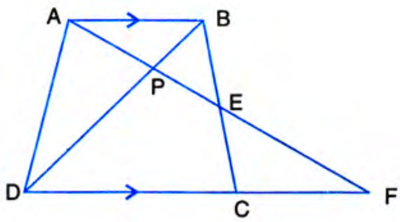
EXERCISE 15(E)

1. In the following figure, XY is parallel to BC, AX = 9 cm, XB = 4·5 cm and BC = 18 cm.

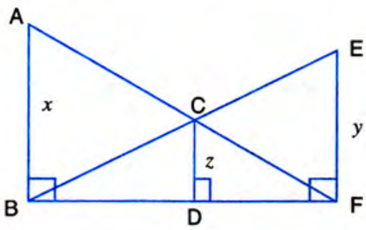


Find :
(i) $\frac{AY}{YC}$ (ii) $\frac{YC}{AC}$ (iii) XY

2. In the following figure, ABCD to a trapezium with AB // DC. If AB = 9 cm, DC = 18 cm, CF = 13·5 cm, AP = 6 cm and BE = 15 cm, Calculate :
   (i) EC (ii) AF (iii) PE



3. In the following figure, AB, CD and EF are perpendicular to the straight line BDF.



If AB = x and, CD = z unit and EF = y unit, prove that : $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

Let BD = a and DF = b. In ΔABF, CD is parallel to AB

$\Rightarrow \frac{CD}{AB} = \frac{DF}{BF}$ *i.e.*, $\frac{z}{x} = \frac{b}{a+b}$.

And, in Δ BEF, CD is parallel to EF

$\Rightarrow \frac{CD}{EF} = \frac{BD}{BF}$ *i.e.* $\frac{z}{y} = \frac{a}{a+b}$.

$\therefore \frac{z}{x} + \frac{z}{y} = \frac{b}{a+b} + \frac{a}{a+b} = 1$

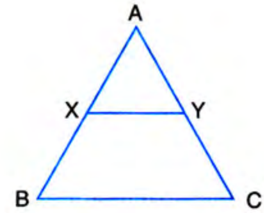
$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

4. Triangle ABC is similar to triangle PQR. If AD and PM are corresponding medians of the two triangles, prove that : $\frac{AB}{PQ} = \frac{AD}{PM}$.

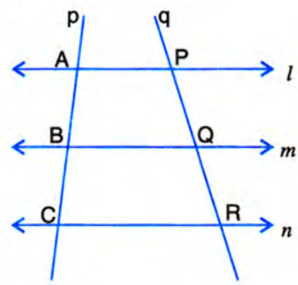
5. Triangle ABC is similar to triangle PQR. If AD and PM are altitudes of the two triangles, prove that : $\frac{AB}{PQ} = \frac{AD}{PM}$.

6. Triangle ABC is similar to triangle PQR. If bisector of angle BAC meets BC at point D and bisector of angle QPR meets QR at point M, prove that : $\frac{AB}{PQ} = \frac{AD}{PM}$.

7. In the following figure, ∠AXY = ∠AYX. If $\frac{BX}{AX} = \frac{CY}{AY}$, show that triangle ABC is isosceles.



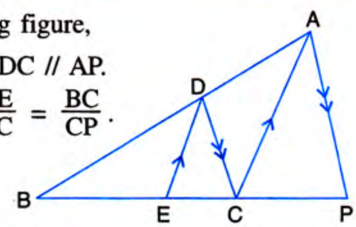
8. In the following diagram, lines l, m and n are parallel to each other. Two transversals p and q intersect the parallel lines at points A, B, C and P, Q, R as shown.



Prove that : $\frac{AB}{BC} = \frac{PQ}{QR}$

Join A and R. Let AR meets BQ at point D.

9. In the following figure, DE // AC and DC // AP. Prove that : $\frac{BE}{EC} = \frac{BC}{CP}$.



In Δ ACB, DE // AC

$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC}$ I

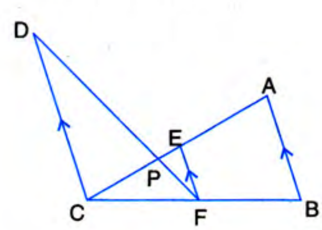
In Δ BAP, DC // AP

$\Rightarrow \frac{BD}{DA} = \frac{BC}{CP}$ II

From I and II, we get : $\frac{BE}{EC} = \frac{BC}{CP}$

10. In the figure given below, AB // EF // CD. If AB = 22·5 cm, EP = 7·5 cm, PC = 15 cm and DC = 27 cm.

Calculate : (i) EF (ii) AC



11. In quadrilateral ABCD, its diagonals AC and BD intersect at point O such that

$$\frac{OC}{OA} = \frac{OD}{OB} = \frac{1}{3}.$$

Prove that :

(i) Δ OAB ~ Δ OCD

(ii) ABCD is a trapezium.

Further if CD = 4·5 cm; find the length of AB.

12. In triangle ABC, angle A is obtuse and AB = AC. P is any point in side BC. PM $\perp$ AB and PN $\perp$ AC.

Prove that : P M × PC = PN × PB

13. In triangle ABC, AB = AC = 8 cm, BC = 4 cm and P is a point in side AC such that AP = 6 cm. Prove that Δ BPC is similar to Δ ABC. Also, find the length of BP.

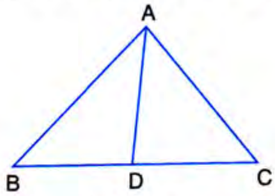
In ΔABC, $\frac{AB}{BC} = \frac{8}{4}$ = 2, in Δ BPC, $\frac{BC}{CP} = \frac{4}{2}$ = 2 and $\angle$ABC = $\angle$C. Therefore, by SAS, Δ ABC is similar to Δ BPC.

14. In ΔABC, $\angle$ABC = $\angle$DAC, AB = 8 cm, AC = 4 cm and AD = 5 cm.

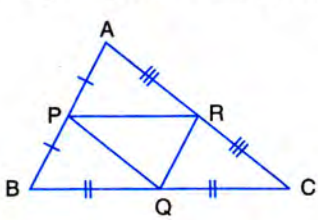
(i) Prove that ΔACD is similar to ΔBCA.

(ii) Find BC and CD.

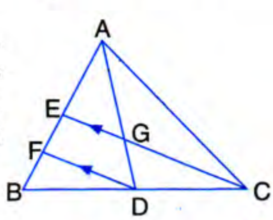
(iii) Find area of ΔACD : area of ΔABC. **[2014]**



15. In the given triangle P, Q and R are the mid-points of sides AB, BC and AC respectively. Prove that triangle PQR is similar to triangle ABC.



16. In the following figure, AD and CE are medians of Δ ABC. DF is drawn parallel to CE.

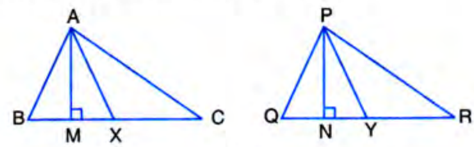


Prove that :

(i) EF = FB,

(ii) AG : GD = 2 : 1.

17. In the given figure, triangle ABC is similar to triangle PQR. AM and PN are altitudes whereas AX and PY are medians.



Prove that : $\frac{AM}{PN} = \frac{AX}{PY}$

18. The two similar triangles are equal in area. Prove that the triangles are congruent.

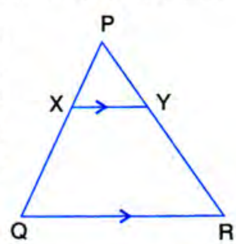
19. The ratio between the altitudes of two similar triangles is 3 : 5; write the ratio between their :

(i) medians. (ii) perimeters. (iii) areas.

20. The ratio between the areas of two similar triangles is 16 : 25. Find the ratio between their :

(i) perimeters. (ii) altitudes. (iii) medians.

21. The given figure shows a triangle PQR in which XY is parallel to QR. If PX : XQ = 1 : 3 and QR = 9 cm, find the length of XY.



Further, if the area of Δ PXY = x cm^2; find, in terms of x, the area of :

(i) triangle PQR. (ii) trapezium XQRY.

22. On a map, drawn to a scale of 1 : 20000, a rectangular plot of land ABCD has AB = 24 cm and BC = 32 cm. Calculate :

(i) the diagonal distance of the plot in kilometre.

(ii) the area of the plot in sq. km.

23. The dimensions of the model of a multistoreyed building are 1 m by 60 cm by 1·20 m. If the scale factor is 1 : 50, find the actual dimensions of the building.

Also, find :

(i) the floor area of a room of the building, if the floor area of the corresponding room in the model is 50 sq. cm.

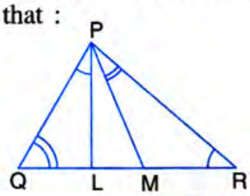
(ii) the space (volume) inside a room of the model, if the space inside the corresponding room of the building is 90 m^3.

24. In a triangle PQR, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$. Prove that :

(i) Δ PQL ~ Δ RPM

(ii) QL × RM = PL × PM

(iii) PQ^2 = QR × QL



[2003]

25. In Δ ABC, $\angle ACB = 90°$ and CD $\perp$ AB.

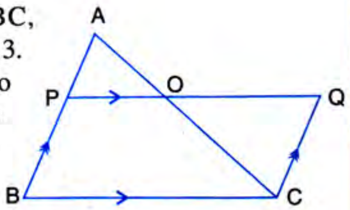
Prove that : $\frac{BC^2}{AC^2} = \frac{BD}{AD}$.

26. A triangle ABC with AB = 3 cm, BC = 6 cm and AC = 4 cm is enlarged to Δ DEF such that the longest side of Δ DEF = 9 cm. Find the scale factor and hence, the lengths of the other sides of Δ DEF.

27. Two isosceles triangles have equal vertical angles. Show that the triangles are similar.

If the ratio between the areas of these two triangles is 16 : 25, find the ratio between their corresponding altitudes.

28. In Δ ABC, AP : PB = 2 : 3. PO is parallel to BC and is extended to Q so that CQ is parallel to BA.

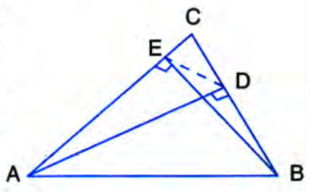


Find :

(i) area Δ APO : area Δ ABC.

(ii) area Δ APO : area Δ CQO.

29. The following figure shows a triangle ABC in which AD and BE are perpendiculars to BC and AC respectively.



Show that :

(i) Δ ADC ~ Δ BEC

(ii) CA × CE = CB × CD

(iii) Δ ABC ~ Δ DEC

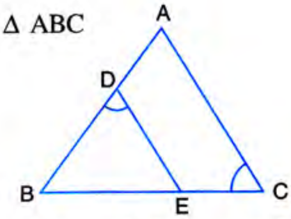
(iv) CD × AB = CA × DE

30. In the given figure, ABC is a triangle with $\angle EDB = \angle ACB$. Prove that Δ ABC ~ Δ EBD.

If BE = 6 cm, EC = 4 cm, BD = 5 cm and area of Δ BED = 9 cm^2. Calculate the :

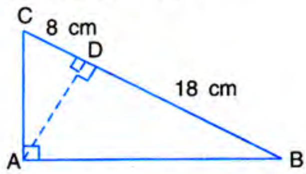
(i) length of AB

(ii) area of Δ ABC



[2010]

31. In the given figure, ABC is a right angled triangle with $\angle BAC = 90°$.

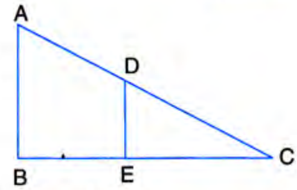


(i) Prove that : ΔADB ~ ΔCDA.

(ii) If BD = 18 cm and CD = 8 cm, find AD.

(iii) Find the ratio of the area of ΔADB is to area of ΔCDA. **[2011]**

32. In the given figure, AB and DE are perpendiculars to BC.

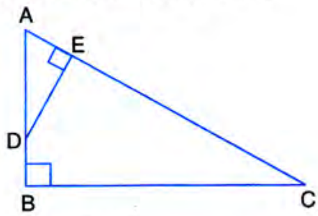


(i) Prove that : ΔABC ~ ΔDEC

(ii) If AB = 6 cm, DE = 4 cm and AC = 15 cm. Calculate CD.

(iii) Find the ratio of the area of a ΔABC : area of ΔDEC. **[2013]**

33. ABC is a right angled triangle with $\angle ABC = 90°$. D is any point on AB and DE is perpendicular to AC. Prove that :



(i) ΔADE ~ ΔACB.

(ii) If AC = 13 cm, BC = 5 cm and AE = 4 cm. Find DE and AD.

(iii) Find, area of ΔADE : area of quadrilateral BCED. **[2015]**

34. Given : AB // DE and BC // EF. Prove that :

(i) $\frac{AD}{DG} = \frac{CF}{FG}$

(ii) Δ DFG ~ Δ ACG.