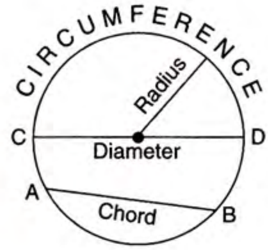
17 Circles

17.1 Introduction :

A circle is defined as the figure (closed curve) obtained by joining all those points in a plane which are at a *constant distance* from a fixed point in the same plane.

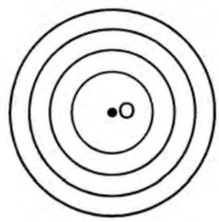


Infact, a circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point in the same plane always remains constant.

The *fixed point* is called the ***centre*** of the circle and the *constant distance* is called the ***radius*** of the circle.

The *perimeter* of the circle is called its ***circumference.***

1. *Concentric circles :*



Two or more circles are said to be concentric if they have same centre and different radii.

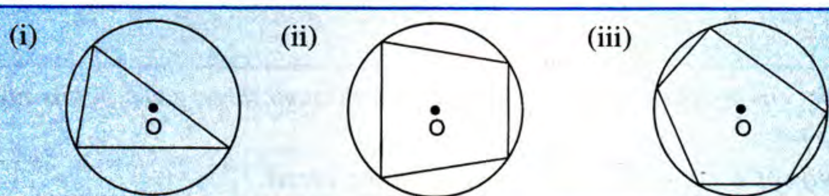
In the adjoining figure, O is the centre of each circle drawn; so the circles are called ***concentric circles***.

2. *Equal circles :*

Circles are said to be equal or congruent if they have equal radii.

3. *Circumscribed circle :*

A circle that passes through all the vertices of a polygon is called the ***circumscribed circle.*** The centre of circumscribed circle is called ***circumcentre*** and the polygon is called ***incribed polygon***. See below :

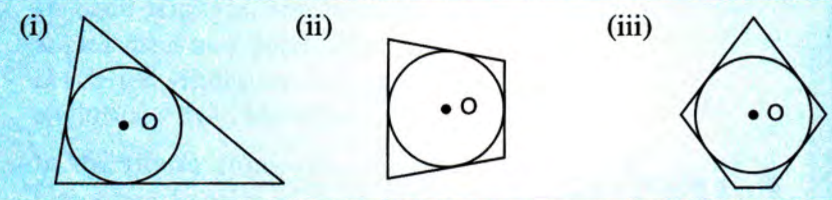


and so on.

4. *Inscribed circle :*

A circle that touches all the sides of a polygon is called the ***inscribed circle*** (or, in-circle) of the polygon.

The centre of inscribed circle is called ***incentre*** and the polygon is called ***circumscribed polygon***. See below :



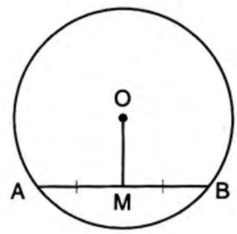
and so on.

5. *Chord :*

The line segment, joining any two points on the circumference of the circle, is called a ***chord***.

A *chord,* which passes through the centre of the circle is called ***diameter***, and is the *largest chord* of the circle.

6. *A straight line drawn from the centre of a circle to bisect a chord, which is not a diameter, is at right angles to the chord.*

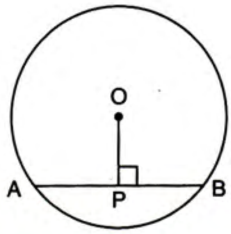


In the given figure, line OM, drawn from the centre O to bisect the chord AB, is perpendicular to AB.

i.e. $AM = BM \Rightarrow OM \perp AB$

$\Rightarrow \angle OMA = \angle OMB = 90°$

7. *The perpendicular to a chord, from the centre of the circle, bisects the chord.*



In the given figure, O is the centre of the circle and OP is perpendicular to the chord AB.

$\Rightarrow AP = BP$

Remember :

Greater is the size of a chord, smaller is its distance from the centre and vice-versa.

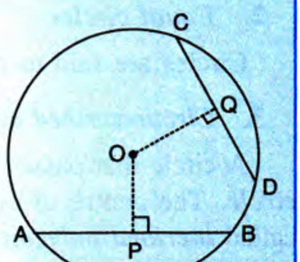
The adjoining figure shows a circle with centre O. OP is perpendicular to chord AB and OQ is perpendicular to chord CD.

Since, chord AB is greater than chord CD

$\Rightarrow$ AB is at smaller distance from the centre as compared to CD

i.e. $OP < OQ$.

Conversely, as $OP < OQ \Rightarrow AB > CD$.



8. *There is one circle, and only one, which passes through three given points not in a straight line.*

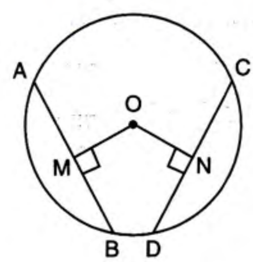
9. *Equal chords of a circle are equidistant from the centre.*

10. *Chords of a circle, equidistant from the centre of the circle, are equal.*

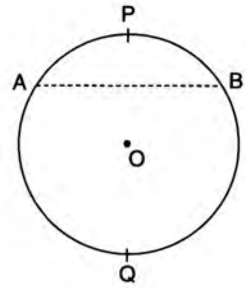
In the given figure,

(i) if $AB = CD \Rightarrow OM = ON$

and, (ii) if $OM = ON \Rightarrow AB = CD$



17.2 Arc and its types :



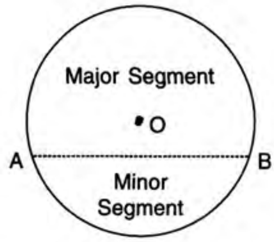
*An **arc** is a part of the circumference of a circle.*

A chord divides the circumference of a circle into two parts and each part is called ***an arc***.

In the figure, given alongside, chord AB divides the circumference into two unequal arcs APB and AQB.

The arc APB, which is *less than* the semi-circle, is called ***minor arc*** and the arc AQB, which is *greater than* the semi-circle, is called ***major arc***.

17.3 Segment and relation between arcs and segments :

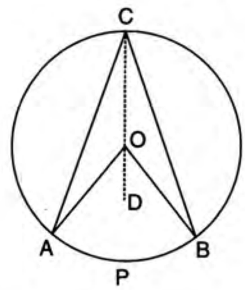


A *segment* is the part of a circle bounded by an arc and a chord.

Theorem 5

The angle which, an arc of a circle subtends at the centre is double that which it subtends at any point on the remaining part of the circumference.

Given *:* A circle with centre O. Arc APB subtends angle AOB at the centre and angle ACB at point C on the remaining circumference.



To Prove *:* $\angle AOB = 2\angle ACB$.

Construction *:* Join CO and produce it to a point D.

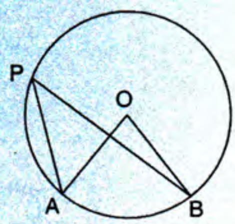
Proof :

	Statement	***Reason***
	In Δ AOC;	
1.	$OA = OC$	Radii of the same circle.
2.	$\therefore \angle OAC = \angle OCA$	Angles opposite to equal sides of a Δ are equal.
3.	$\angle AOD = \angle OAC + \angle OCA$	Exterior angle of a Δ = sum of its interior opposite angles.
	$= \angle OCA + \angle OCA$	From (2) : $\angle OAC = \angle OCA$
	$= 2\angle OCA$	
4.	Similarly, in Δ BOC,	
	Ext. $\angle BOD = 2\angle OCB$	
5.	$\mathbf{\angle AOB} = \angle AOD + \angle BOD$	
	$= 2\angle OCA + 2\angle OCB$	From (3) and (4)
	$= 2(\angle OCA + \angle OCB)$	
	$= \mathbf{2\angle ACB}$	

Hence Proved.

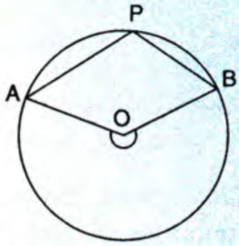
Similarly, consider the following figures :

(i)



$\angle AOB = 2\angle APB$

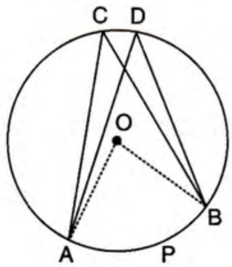
(ii)



Reflex $\angle AOB = 2\angle APB$

Theorem 6

Angles in the same segment of a circle are equal.



Given : A circle with centre O. Angle ACB and angle ADB are in the same segment.

To Prove : $\angle ACB = \angle ADB$.

Construction : Join OA and OB.

Proof :

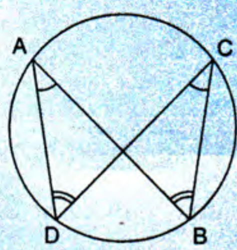
	Statement	***Reason***
1.	Arc APB subtends angle AOB at the centre and angle ACB at point C of the remaining circumference.	
	$\therefore \quad \angle AOB = 2\angle ACB$	Angle at the centre is twice the angle at remaining circumference.
2.	Similarly, $\angle AOB = 2\angle ADB$	" "
3.	$\therefore \quad$ **$\angle ACB = \angle ADB$**	From (1) and (2)

Hence Proved.

Similarly, in the adjoining figure :

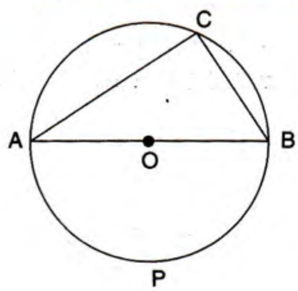
(i) $\angle DAB = \angle DCB$ [Angles in the same segment]

(ii) $\angle ADC = \angle ABC$ [Angles in the same segment]



Theorem 7

The angle in a semi-circle is a right angle.



Given : A circle with centre O. AB is a diameter and ACB is the angle of semi-circle.

To Prove : $\angle ACB = 90°$.

Proof :

Statement	*Reason*
1. Arc APB subtends $\angle AOB$ at the centre and $\angle ACB$ at pt. C of remaining circumference.	
$\therefore \quad \angle AOB = 2\angle ACB$	Angle at the centre is twice the angle at remaining circumference.
2. $\angle AOB = 180°$	AOB is a straight line.
3. $2\angle ACB = 180°$	From (1) and (2)
$\Rightarrow \quad \mathbf{\angle ACB = 90°}$	

Hence Proved.

17.4 Cyclic Properties :

When a quadrilateral is inscribed in a circle *i.e.* the vertices of the quadrilateral lie on the circumference of a circle; the quadrilateral is called a ***cyclic quadrilateral***.

The points, which lie on the circumference of the same circle, are called ***concyclic points***.

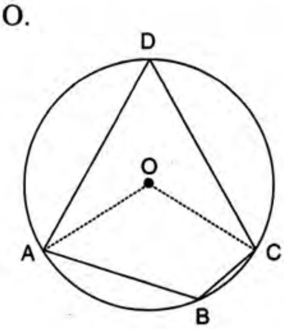
Theorem 8

The opposite angles of a cyclic quadrilateral (quadrilateral inscribed in a circle) are supplementary.

Given : A quadrilateral ABCD inscribed in a circle with centre O.

To Prove : $\angle ABC + \angle ADC = 180°$
and, $\angle BAD + \angle BCD = 180°$.

Construction : Join OA and OC.



Proof :

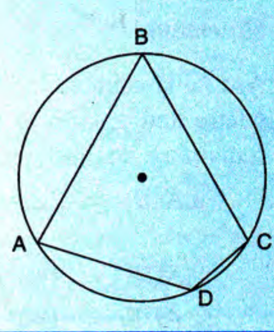
Statement	*Reason*
1. Arc ABC subtends angle AOC at the centre and angle ADC at point D of the remaining circumference.	
$\therefore \angle AOC = 2\angle ADC$	Angle at the centre is twice the angle at remaining circumference.
$\Rightarrow \angle ADC = \frac{1}{2}\angle AOC$	
Similarly,	
2. $\angle ABC = \frac{1}{2}$ reflex $\angle AOC$	
3. $\mathbf{\angle ABC + \angle ADC}$	
$= \frac{1}{2}$ (reflex $\angle AOC + \angle AOC$)	From (1) and (2)
$= \frac{1}{2} \times 360° = \mathbf{180°}$	Reflex $\angle AOC + \angle AOC = 360°$

Similarly, $\mathbf{\angle BAD + \angle BCD = 180°}$

Hence Proved.

Remember :

1. If the opposite angles of a quadrilateral are supplementary, the quadrilateral is cyclic.
2. Angle of major segment is acute and angle of minor segment is obtuse.
   In the adjoining figure, ∠ABC is the angle of major segment, so it is acute *i.e.* ∠ABC is less than 90°. In the same way, ∠ADC is the angle of minor segment, so it is obtuse *i.e.* angle ADC is greater than 90°.

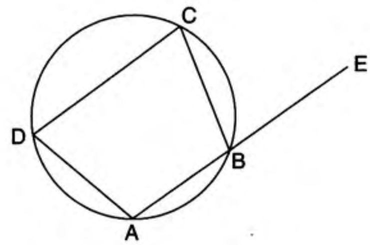


Theorem 9

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Given : A cyclic quadrilateral ABCD whose side AB is produced to a point E.

To Prove : Ext. ∠CBE = ∠ADC



Proof :

	Statement	***Reason***
1.	∠ABC + ∠CBE = 180°	∠ABC + ∠CBE = ∠ABE = 180°
2.	∠ABC + ∠ADC = 180°	Opp. angles of a cyclic quadrilateral are supplementary.
∴	∠ABC + ∠CBE = ∠ABC + ∠ADC	From (1) and (2)
⇒	**∠CBE = ∠ADC.**	

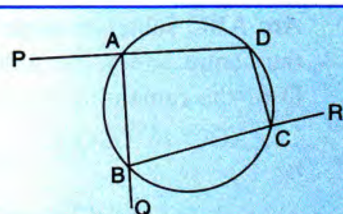
Hence Proved.

Similarly, in the adjoining figure :

(i) ∠PAB = ∠BCD,

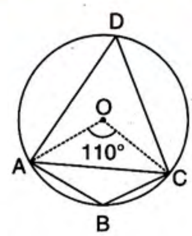
(ii) ∠QBC = ∠ADC,

(iii) ∠RCD = ∠BAD and so on.



1 In the adjoining figure; ∠AOC = 110°; calculate :

(i) ∠ADC (ii) ∠ABC (iii) ∠OAC.



Solution :

(i) **∠ADC** = $\frac{1}{2}$∠AOC [Angle at centre is twice the angle at remaining circumference]

= $\frac{1}{2}$ × 110° = **55°** **Ans.**

(ii) $\angle ABC + \angle ADC = 180°$ [Opp. angles of a cyclic quadrilateral are supplementary]

$\therefore \quad \angle ABC + 55° = 180°$

$\Rightarrow \quad \angle ABC = 125°$ **Ans.**

(iii) In Δ AOC,

$OA = OC$ [Radii of the same circle]

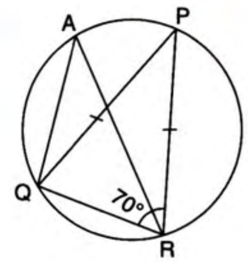
$\therefore \quad \angle OAC = \angle OCA$ [Angles opposite to equal sides]

Since, $\angle OAC + \angle OCA + \angle AOC = 180°$ [Sum of angles of a $\Delta = 180°$]

$\Rightarrow \angle OAC + \angle OAC + 110° = 180°$ [$\angle OCA = \angle OAC$]

$\Rightarrow \quad \angle OAC = 35°$ **Ans.**

2 In the adjoining figure, PQ = PR and $\angle PRQ = 70°$. Find $\angle QAR$.



Solution :

In ΔPQR,

$PQ = PR$ [Given]

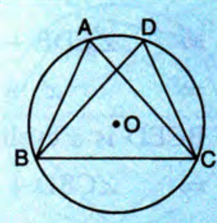
$\therefore \angle PQR = \angle PRQ = 70°$ [In a Δ, angles opposite to equal sides are equal]

$\therefore \angle QPR = 180° - (70° + 70°) = 40°$ [Sum of angles of a $\Delta = 180°$]

Now, $\angle QAR = \angle QPR$ [Angles of same segment are equal]

$= 40°$ **Ans.**

3 The given figure shows a circle through the points A, B, C and D. If $\angle BAC = 67°$, find : $\angle DBC + \angle DCB$.



Solution :

$\angle BDC = \angle BAC = 67°$ [Angles of the same segment are equal]

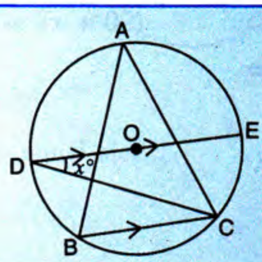
In ΔDBC,

$\angle BDC + \angle DBC + \angle DCB = 180°$

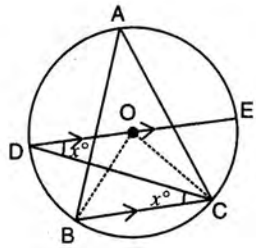
$\Rightarrow \quad 67° + \angle DBC + \angle DCB = 180°$

$\Rightarrow \quad \angle DBC + \angle DCB = 180° - 67° = 113°$ **Ans.**

4 In the given figure, BC//DE and O is the centre of the circle. If $\angle CDE = x°$; find, in terms of $x°$, the value of $\angle BAC$.



Solution :



Since, BC//DE and DC is a transversal,

$\therefore \quad \angle BCD = \angle CDE = x°$ [Alternate angles]

Now join OB and OC.

Since, angle at the centre is twice the angle at remaining circumference.

$\therefore \quad \angle COE = 2\angle CDE = 2x°$

and, $\quad \angle BOD = 2\angle BCD = 2x°$

DOE is a straight line (diameter)

$\therefore \quad \angle DOB + \angle BOC + \angle COE = 180°$

$\Rightarrow \quad 2x° + \angle BOC + 2x° = 180°$

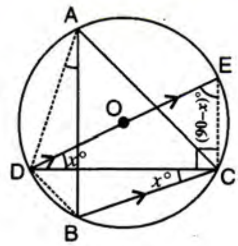
$\Rightarrow \quad \angle BOC = (180 - 4x)°$

Also, $\quad \angle BOC = 2\angle BAC$

$\Rightarrow \quad (180° - 4x)° = 2\angle BAC$

and, $\quad \angle BAC = \dfrac{(180-4x)°}{2} = (90 - 2x)°$ **Ans.**

Second method :



Join CE and BD

$\angle DCE = 90°$ [Angle of semi-circle]

$\Rightarrow \quad \angle CDE + \angle CED = 90°$

$\Rightarrow \quad x° + \angle CED = 90°$ *i.e.* $\angle CED = (90 - x)°$

BCED is a cyclic quadrilateral

$\Rightarrow \quad \angle CBD + \angle CED = 180°$ [Opp. angles of cyclic quadrilateral are supplementary]

$\Rightarrow \quad \angle CBD + (90 - x)° = 180°$

$\Rightarrow \quad \angle CBD = (180 - 90 + x)°$

$= (90 + x)°$

Also, $\quad \angle BCD = \angle CDE = x°$ [Alternate angles]

$\therefore$ In ΔCBD,

$\angle CBD + \angle BCD + \angle BDC = 180°$

$\Rightarrow \quad (90 + x)° + x° + \angle BDC = 180°$

$\Rightarrow \quad \angle BDC = (180 - 90 - x - x)°$

$= (90 - 2x)°$

$\therefore \quad \angle BAC = \angle BDC$ [Angles of the same segment]

$= (90 - 2x)°$

Third method :

Join AD and CE

Show that $\angle CED = (90 - x)^\circ$ and

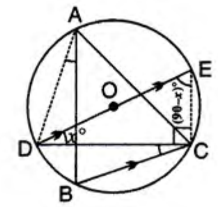
$\angle BCD = x^\circ$

$\angle BAD = \angle BCD = x^\circ$

[Angles of same segment]

$\angle DAC = \angle DEC = (90 - x)^\circ$

[Angles of same segment]

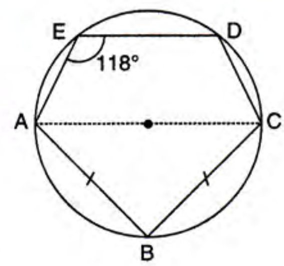


$\Rightarrow \angle BAD + \angle BAC = (90 - x)^\circ$ [$\angle BAD + \angle BAC = \angle DAC$]

$\Rightarrow x^\circ + \angle BAC = (90 - x)^\circ$

$\Rightarrow$ **$\angle BAC = (90 - 2x)^\circ$** **Ans.**

5 In the adjoining figure; AC is a diameter of the circle. AB = BC and $\angle AED = 118°$. Calculate :
(i) $\angle DEC$ (ii) $\angle DAB$



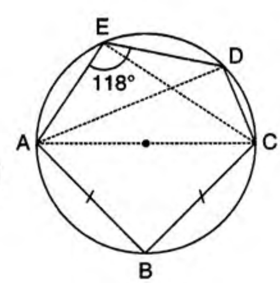
Solution :

(i) Join EC.

$\angle AEC = 90°$ [Angle of semi-circle; as AC is diameter]

$\angle DEC$ $= 118° - 90° =$ **$28°$** **Ans.**

(ii) Join AD.



$\angle DAC = \angle DEC = 28°$ [Angles of same segment are equal]

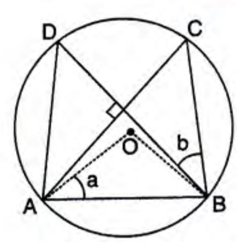
$\angle ABC = 90°$ [Angle of semi-circle = 90°]

$\angle BAC = \angle BCA = 45°$ [Given, AB = BC]

$\therefore$ **$\angle DAB$** $= \angle DAC + \angle BAC$

$= 28° + 45° =$ **$73°$** **Ans.**

6 In the adjoining figure; O is centre of the circle, chords AC and BD are perpendicular to each other, $\angle OAB = a$ and $\angle DBC = b$. Show that $a = b$.



Solution :

In Δ AOB,

OA = OB [Radii of same circle]

$\therefore \angle OBA = \angle OAB = a$ [Angles opposite to equal sides]

$\therefore \angle AOB = 180° - (a + a)$

$= 180° - 2a$

$\angle ACB = \frac{1}{2}\angle AOB$ [Angle at the centre is twice the angle at remaining circumference]

$= \frac{1}{2}(180° - 2a) = 90° - a$

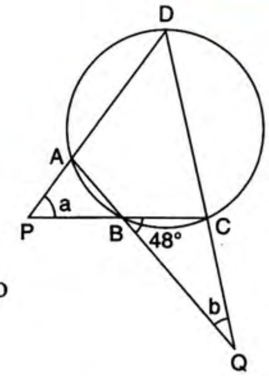
Given chords AC and BD are perpendicular to each other,

$\therefore \quad \angle ACB + \angle DBC = 90°$

$\Rightarrow \quad 90° - a + b = 90°$

$\Rightarrow \quad \mathbf{a = b}$ **Ans.**

7 In the adjoining figure; ABCD is a cyclic quadrilateral, $\angle CBQ = 48°$ and $a = 2b$. Calculate the numerical value of b.



Solution :

In Δ BCQ,

$\angle DCB = 48° + b$ [Exterior angle = sum of two interior opp. angles]

In Δ APB,

$\angle ABP = \angle CBQ = 48°$ [Vertically opposite angles are equal]

$\therefore \angle DAB = \angle ABP + a$ [Exterior angle = sum of two interior opp. angles]

$= 48° + a$

Now, in cyclic quadrilateral ABCD,

$\angle DCB + \angle DAB = 180°$ [Opp. angles of a cyclic quad. are supplementary]

$\Rightarrow \quad 48° + b + 48° + a = 180°$

$\Rightarrow \quad a + b = 84°$

$\Rightarrow \quad 2b + b = 84°$ [Given, $a = 2b$]

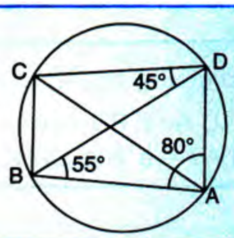
$\Rightarrow \quad \mathbf{b = 28°}$ **Ans.**

8 In the given figure, $\angle BAD = 80°$, $\angle ABD = 55°$ and $\angle BDC = 45°$. Find :

(i) $\angle BCD$

(ii) $\angle ADB$

Hence, show that AC is a diameter. **[2006 type]**



Solution :

(i) Since, ABCD is a cyclic quadrilateral and opposite angles of a cyclic quadrilateral are supplementary :

$\therefore \quad \angle BAD + \angle BCD = 180° \quad \Rightarrow 80° + \angle BCD = 180°$

$\Rightarrow \quad \mathbf{\angle BCD = 100°}$ **Ans.**

(ii) Since, the sum of the angles of a triangle is 180°

$\therefore$ In Δ ABD, $\angle ADB + 55° + 80° = 180°$

$\angle ADB = 180° - 135° = 45°$ **Ans.**

AC will be a diameter of the circle, if $\angle ADC = 90°$ or $\angle ABC = 90°$.

$\angle ADC = \angle ADB + \angle BDC$

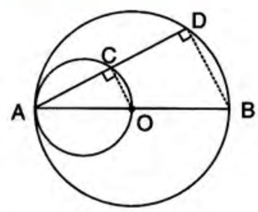
$= 45° + 45° = 90°$

$\Rightarrow$ $\angle ADC$ is the angle of semi-circle and so **AC is a diameter of the circle.**

9 In a circle, with centre O, a diameter AB and a chord AD are drawn. Another circle is drawn with AO as diameter to cut AD at C.
Prove that : BD = 2 × OC **[2005]**

Solution :

According to the given statement, the figure will be as shown alongside :



Join OC and BD.

Clearly, $\angle ACO = \angle ADB = 90°$ [Angles of semi-circles]

OA is radius and AB is diameter $\Rightarrow$ $OA = \frac{1}{2}AB$

i.e. $\frac{OA}{AB} = \frac{1}{2}$ .... (I)

OC is $\perp$ to chord AD $\Rightarrow$ $AC = \frac{1}{2}AD$ [$\perp$ from centre bisects the chord]

i.e. $\frac{AC}{AD} = \frac{1}{2}$ .... (II)

In Δ AOC and Δ ABD,

$\frac{OA}{AB} = \frac{AC}{AD}$ and $\angle A$ is common

$\Rightarrow$ $\Delta AOC \sim \Delta ABD$ [By S.A.S.]

$\Rightarrow$ $\frac{OC}{BD} = \frac{OA}{AB}$ [Corresponding sides of similar Δs are proportional]

$\Rightarrow$ $\frac{OC}{BD} = \frac{1}{2}$ $\Rightarrow$ **BD = 2 × OC** **Hence Proved.**

Alternative method :

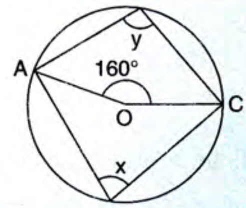
$\because$ $OA = \frac{1}{2}AB \Rightarrow$ O is mid-point of AB

$AC = \frac{1}{2}AD$ [$\perp$ OC bisects the chord AD]

$\therefore$ In Δ ABD, $OC = \frac{1}{2}BD$ [Line joining the mid-points of two sides of a Δ is half of the third side]

$\Rightarrow$ **BD = 2 × OC** **Hence Proved.**

10 In the figure, given alongside, O is the centre of the circle and $\angle AOC = 160°$. Prove that : $3\angle y - 2\angle x = 140°$ [2005]



Solution :

Since angle at centre is twice the angle at remaining circumference

$\therefore$ $\angle AOC = 2x$ *i.e.* $2x = 160°$ and $x = 80°$

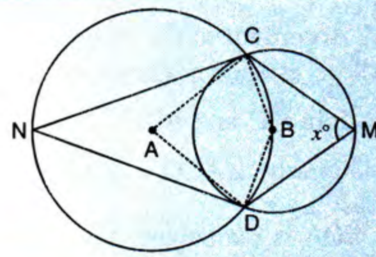
Since, opposite angles of a cyclic-quadrilateral are supplementary

$\therefore$ $x + y = 180°$ *i.e.* $y = 180° - x = 180° - 80° = 100°$

Now $3\angle y - 2\angle x = 3 \times 100° - 2 \times 80°$

$= 300° - 160° = 140°$ **Hence Proved.**

11 Two unequal circles with centres A and B intersect each other at points C and D. The centre B of the smaller circle lies on the circumference of the bigger circle with centre A. If $\angle CMD = x°$, find in terms of x, the measure of angle DAC.



Solution :

Since the angle at centre is twice the angle at remaining circumference, therefore in circle with centre B,

$\angle CBD = 2\angle CMD = 2 \times x° = 2x°$

Now in the bigger circle, DBCN is a cyclic quadrilateral and we know the opposite angles of a cyclic quadrilateral are supplementary.

$\therefore$ $\angle DNC + \angle CBD = 180°$

$\Rightarrow$ $\angle DNC + 2x° = 180°$

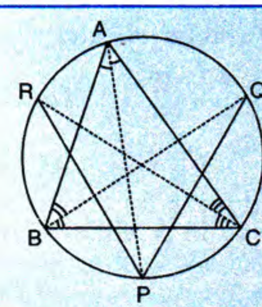
$\Rightarrow$ $\angle DNC = 180° - 2x°$

Again, angle at the centre is double the angle at remaining circumference.

$\therefore$ $\angle DAC = 2\angle DNC$

$= 2(180° - 2x°) = 360° - 4x°$ **Ans.**

12 The given figure shows a triangle ABC with $\angle BAC = 56°$ and $\angle ABC = 64°$. Bisectors of angles A, B and C meet the circumcircle of the Δ ABC at points P, Q and R respectively. Find the measure of $\angle QPR$.



Solution :

In given Δ ABC,

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$\Rightarrow \quad 64^\circ + 56^\circ + \angle ACB = 180^\circ$$

$$\Rightarrow \quad \angle ACB = 60^\circ$$

$\angle QPA = \angle QBA$ [Angles of same segment]

$= \frac{1}{2}\angle ABC$ [Given BQ bisects $\angle ABC$]

$= \frac{1}{2} \times 64^\circ = 32^\circ$

$\angle RPA = \angle RCA$ [Angles of same segment]

$= \frac{1}{2}\angle ACB$ [Given CR bisects $\angle ACB$]

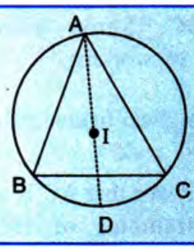
$= \frac{1}{2} \times 60^\circ = 30^\circ$

$\therefore$ **$\angle QPR$** $= \angle QPA + \angle RPA$

$= 32^\circ + 30^\circ =$ **62°** **Ans.**

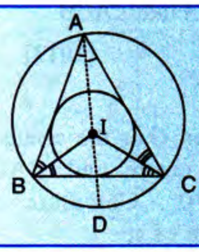
13 In the given figure, I is the incentre of triangle ABC. AI produced meets the circumcircle of the triangle ABC at point D. If $\angle BAC = 50^\circ$ and $\angle ABC = 70^\circ$, find :

(i) $\angle BCD$ (ii) $\angle ICD$ (iii) $\angle BIC$



Solution :

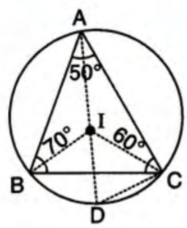
Here, I is incentre of ΔABC means, I is the centre of the circle that touches all the sides of ΔABC and it is the point of intersection of the bisectors of angles of the triangle ABC. That is, IA bisects $\angle BAC$, IB bisects $\angle CBA$ and IC bisects $\angle ACB$.



Join IB, IC and CD.

Since, $\angle BAC = 50^\circ$ and $\angle ABC = 70^\circ$

so, $\angle ACB = 60^\circ$



(i) Since, I is incentre, so IA is the bisector of $\angle BAC$

$\Rightarrow \quad \angle BAD = \frac{1}{2}\angle BAC$ [IA bisects $\angle BAC$]

$= \frac{1}{2} \times 50^\circ = 25^\circ$

$\angle BCD$ $= \angle BAD$ [Angles of the same segment]

$=$ **25°** **Ans.**

(ii) IC bisects ∠ACB

$\Rightarrow \qquad \angle ICB = \frac{1}{2}\angle ACB = \frac{1}{2} \times 60° = 30°$

$\therefore \qquad$ **∠ICD** $= \angle ICB + \angle BCD$

$= 30° + 25° =$ **55°** **Ans.**

(iii) $\because \qquad \angle IBC = \frac{1}{2}\angle ABC$ [IB bisects ∠ABC]

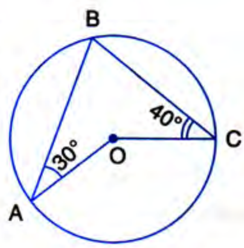
$= \frac{1}{2} \times 70° = 35°$

$\therefore \qquad$ **∠BIC** $= 180° - \angle IBC - \angle ICB$

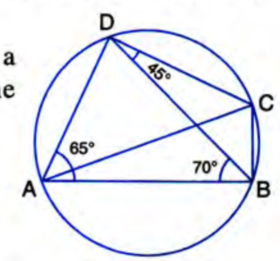
$= 180° - 35° - 30° =$ **115°** **Ans.**

EXERCISE 17(A)

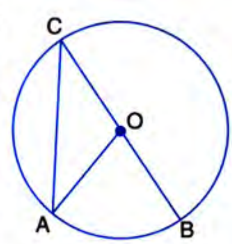
1. In the given figure, O is the centre of the circle. ∠OAB and ∠OCB are 30° and 40° respectively. Find ∠AOC. Show your steps of working.



2. In the given figure, ∠BAD = 65°, ∠ABD = 70°, ∠BDC = 45°
   (i) Prove that AC is a diameter of the circle.
   (ii) Find ∠ACB. **[2013]**

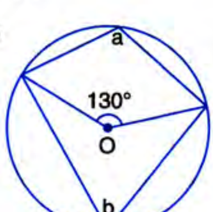


3. Given O is the centre of the circle and ∠AOB = 70°. Calculate the value of :
   (i) ∠OCA,
   (ii) ∠OAC.

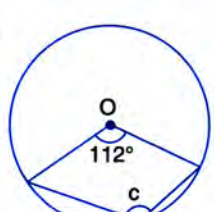


4. In each of the following figures, O is the centre of the circle. Find the values of *a*, *b* and *c*.

(i)

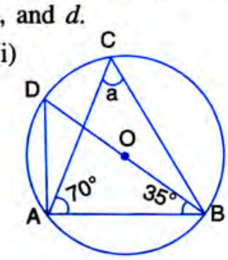


(ii)

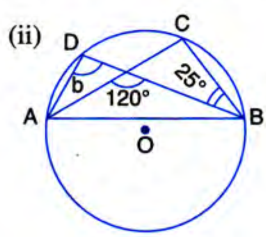


5. In each of the following figures, O is the centre of the circle. Find the values of *a*, *b*, c, and *d*.

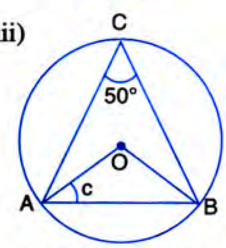
(i)



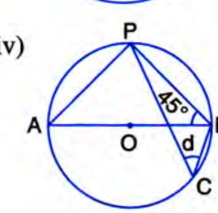
(ii)



(iii)

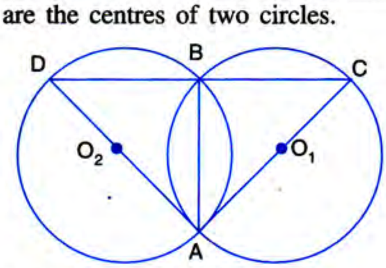


(iv)



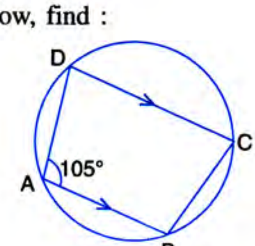
[2007]

6. In the figure, AB is common chord of the two circles. If AC and AD are diameters; prove that D, B and C are in a straight line. O_1 and O_2 are the centres of two circles.



7. In the figure, given below, find :
   (i) ∠BCD,
   (ii) ∠ADC,
   (iii) ∠ABC.

   Show steps of your working.



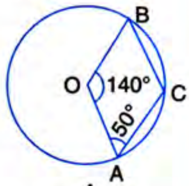
8. In the given figure, O is the centre of the circle. If ∠AOB = 140° and ∠OAC = 50°; find :

(i) ∠ACB,

(ii) ∠OBC,

(iii) ∠OAB,

(iv) ∠CBA.

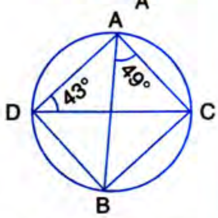


9. Calculate :

(i) ∠CDB,

(ii) ∠ABC,

(iii) ∠ACB.

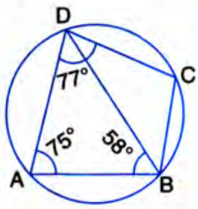


10. In the figure, given below, ABCD is a cyclic quadrilateral in which ∠BAD = 75°; ∠ABD = 58° and ∠ADC = 77°. Find :

(i) ∠BDC,

(ii) ∠BCD,

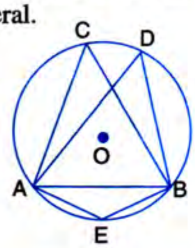
(iii) ∠BCA.



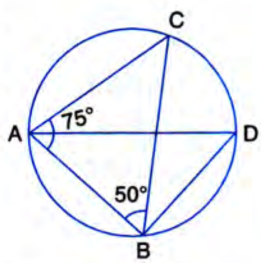
11. In the following figure, O is centre of the circle and Δ ABC is equilateral.

Find :

(i) ∠ADB, (ii) ∠AEB.

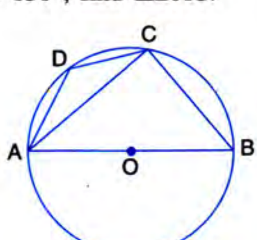


12. Given : ∠CAB = 75° and ∠CBA = 50°. Find the value of ∠DAB + ∠ABD.

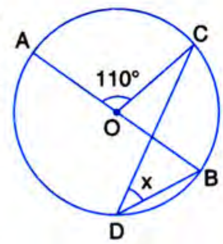


13. ABCD is a cyclic quadrilateral in a circle with centre O.

If ∠ADC = 130°; find ∠BAC.



14. In the figure, given alongside, AOB is a diameter of the circle and ∠AOC = 110°. Find ∠BDC.

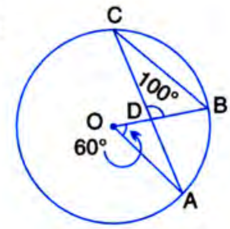


15. In the following figure, O is the centre of the circle,

∠AOB = 60° and

∠BDC = 100°.

Find ∠OBC.

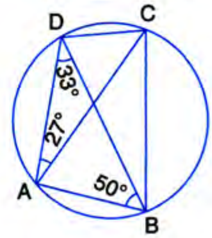


16. ABCD is a cyclic quadrilateral in which ∠DAC = 27°; ∠DBA = 50° and ∠ADB = 33°. Calculate :

(i) ∠DBC,

(ii) ∠DCB,

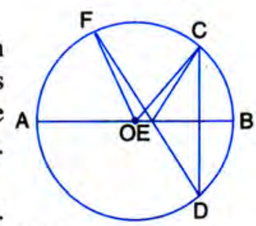
(iii) ∠CAB.



17. In the figure, given alongside, AB is diameter of the circle whose centre is O. Given that :

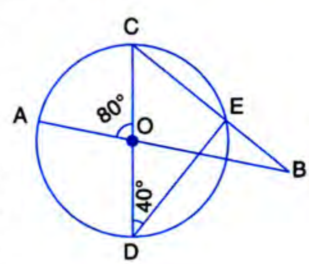
∠ECD = ∠EDC = 32°.

Show that : ∠COF = ∠CEF.



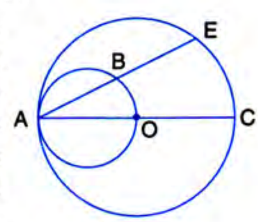
18. In the figure given alongside, AB and CD are straight lines through the centre O of a circle. If ∠AOC = 80° and ∠CDE = 40°, find the number of degrees in :

(i) ∠DCE, (ii) ∠ABC.



19. In the given figure, AC is a diameter of a circle, whose centre is O. A circle is described on AO as diameter. AE, a chord of the larger circle, intersects the smaller circle at B.

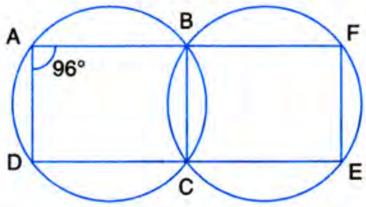
Prove that : AB = BE.



20. In the following figure,

(i) if $\angle BAD = 96°$, find $\angle BCD$ and $\angle BFE$.

(ii) Prove that AD is parallel to FE.

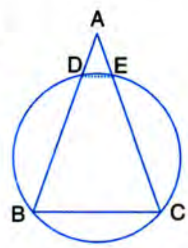


21. Prove that :

(i) the parallelogram, inscribed in a circle, is a rectangle.

(ii) the rhombus, inscribed in a circle, is a square.

22. In the following figure, AB = AC. Prove that DECB is an isosceles trapezium.



23. Two circles intersect at P and Q. Through P diameters PA and PB of the two circles are drawn. Show that the points A, Q and B are collinear.

24. ABCD is a quadrilateral inscribed in a circle, having $\angle A = 60°$; O is the centre of the circle. Show that :

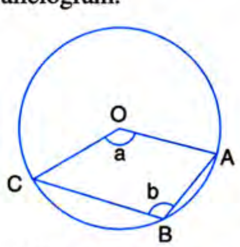
$$\angle OBD + \angle ODB = \angle CBD + \angle CDB.$$

25. The figure given below, shows a circle with centre O.

Given : $\angle AOC = a$ and $\angle ABC = b$.

(i) Find the relationship between a and b.

(ii) Find the measure of angle OAB, if OABC is a parallelogram.

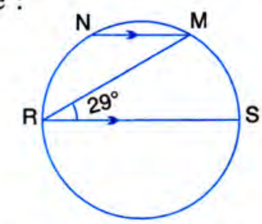


26. Two chords AB and CD intersect at P inside the circle. Prove that the sum of the angles subtended by the arcs AC and BD at the centre O is equal to twice the angle APC.

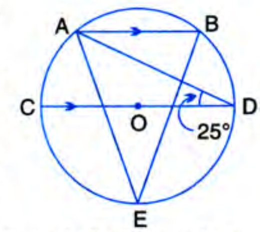
27. In the given figure, RS is a diameter of the circle. NM is parallel to RS and $\angle MRS = 29°$. Calculate :

(i) $\angle RNM$,

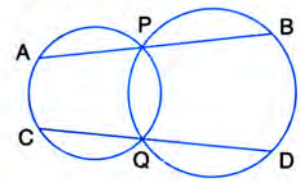
(ii) $\angle NRM$.



28. In the figure, given alongside, AB // CD and O is the centre of the circle. If $\angle ADC = 25°$; find the angle AEB. Give reasons in support of your answer.



29. Two circles intersect at P and Q. Through P, a straight line APB is drawn to meet the circles in A and B. Through Q, a straight line is drawn to meet the circles at C and D. Prove that AC is parallel to BD.



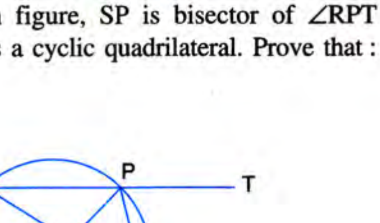
30. ABCD is a cyclic quadrilateral in which AB and DC on being produced, meet at P such that PA = PD. Prove that AD is parallel to BC.

31. AB is a diameter of the circle APBR as shown in the figure. APQ and RBQ are straight lines. Find :

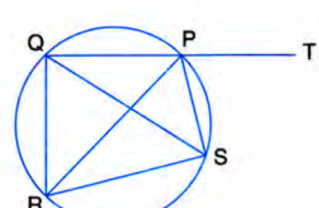
(i) $\angle PRB$,

(ii) $\angle PBR$,

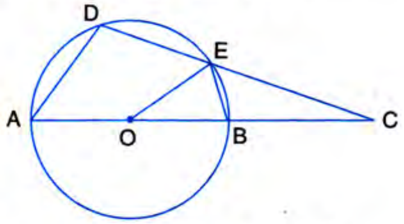
(iii) $\angle BPR$.



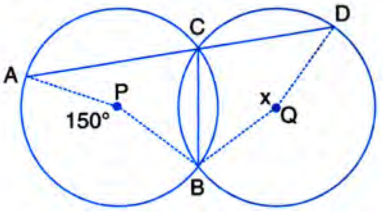
32. In the given figure, SP is bisector of $\angle RPT$ and PQRS is a cyclic quadrilateral. Prove that : SQ = SR.



33. In the figure, O is the centre of the circle, ∠AOE = 150°, ∠DAO = 51°. Calculate the sizes of the angles CEB and OCE.



34. In the figure, given below, P and Q are the centres of two circles intersecting at B and C. ACD is a straight line. Calculate the numerical value of x.



35. The figure shows two circles which intersect at A and B. The centre of the smaller circle is O and lies on the circumference of the larger circle. Given ∠APB = a°.

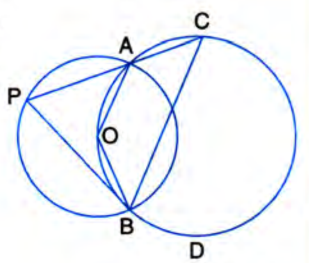
Calculate, in terms of a°, the value of :

(i) obtuse ∠AOB,

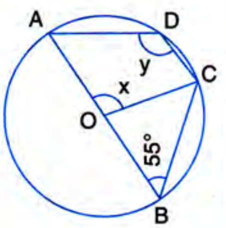
(ii) ∠ACB,

(iii) ∠ADB.

Give reasons for your answers clearly.

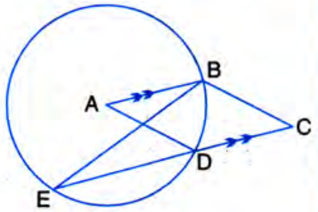


36. In the given figure, O is the centre of the circle and ∠ABC = 55°. Calculate the values of x and y.



37. In the given figure, A is the centre of the circle, ABCD is a parallelogram and CDE is a straight line.

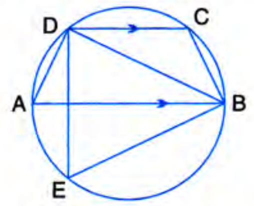
Prove that : ∠BCD = 2∠ABE.



38. ABCD is a cyclic quadrilateral in which AB is parallel to DC and AB is a diameter of the circle. Given ∠BED = 65°; calculate :

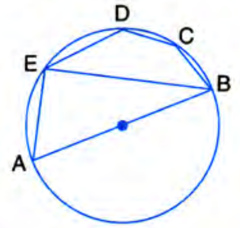
(i) ∠DAB,

(ii) ∠BDC.



39. In the given figure, AB is a diameter of the circle. Chord ED is parallel to AB and ∠EAB = 63°. Calculate :

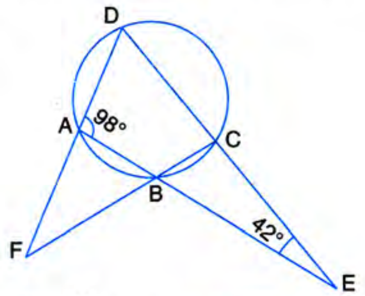
(i) ∠EBA, (ii) ∠BCD.



40. The sides AB and DC of a cyclic quadrilateral ABCD are produced to meet at E; the sides DA and CB are produced to meet at F. If ∠BEC = 42° and ∠BAD = 98°; calculate :

(i) ∠AFB,

(ii) ∠ADC.

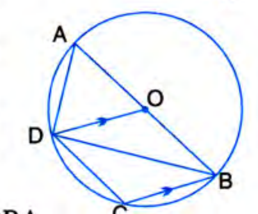


41. In the given figure, AB is a diameter of the circle with centre O. DO is parallel to CB and ∠DCB = 120°. Calculate :

(i) ∠DAB, (ii) ∠DBA,

(iii) ∠DBC, (iv) ∠ADC.

Also, show that the Δ AOD is an equilateral triangle.



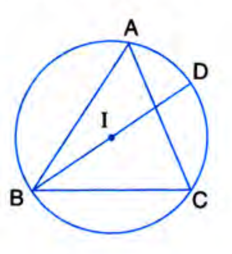
42. In the given figure, I is the incentre of Δ ABC. BI when produced meets the circumcircle of Δ ABC at D. Given ∠BAC = 55° and ∠ACB = 65°; calculate :

(i) ∠DCA,

(ii) ∠DAC,

(iii) ∠DCI,

(iv) ∠AIC.

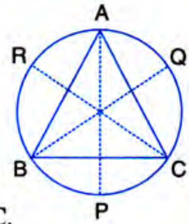


43. A triangle ABC is inscribed in a circle. The bisectors of angles BAC, ABC and ACB meet the circumcircle of the triangle at points P, Q and R respectively. Prove that :

(i) $\angle ABC = 2\angle APQ$,

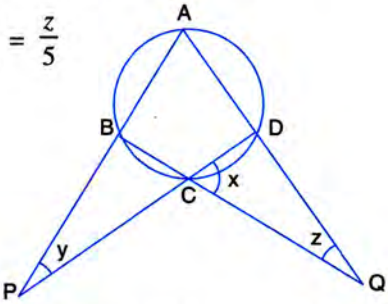
(ii) $\angle ACB = 2\angle APR$,

(iii) $\angle QPR = 90° - \frac{1}{2}\angle BAC$.



44. Calculate the angles x, y and z if :

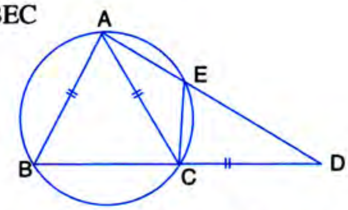
$$\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$$



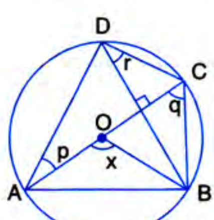
45. In the given figure, AB = AC = CD and $\angle ADC = 38°$. Calculate :

(i) Angle ABC

(ii) Angle BEC



46. In the given figure, AC is the diameter of circle, centre O. Chord BD is perpendicular to AC. Write down the angles p, q and r in terms of x.

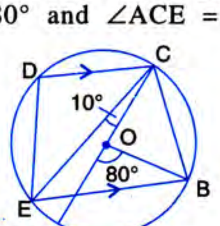


47. In the given figure, AC is the diameter of the circle with centre O. CD and BE are parallel. Angle $\angle AOB = 80°$ and $\angle ACE = 10°$. Calculate :

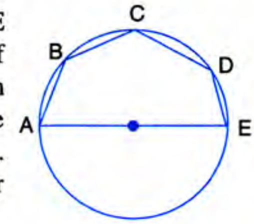
(i) Angle BEC,

(ii) Angle BCD,

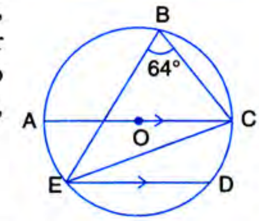
(iii) Angle CED.



48. In the given figure, AE is the diameter of the circle. Write down the numerical value of $\angle ABC + \angle CDE$. Give reasons for your answer.



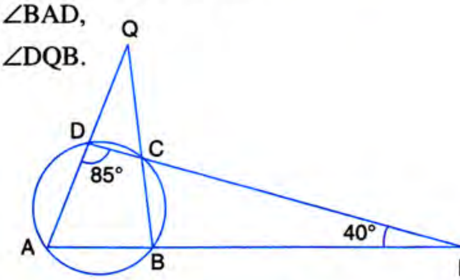
49. In the given figure, AOC is a diameter and AC is parallel to ED. If $\angle CBE = 64°$, calculate $\angle DEC$.



50. Use the given figure to find :

(i) $\angle BAD$,

(ii) $\angle DQB$.



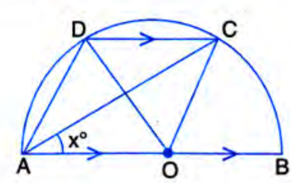
51. In the given figure, AOB is a diameter and DC is parallel to AB. If $\angle CAB = x°$; find (in terms of x) the values of :

(i) $\angle COB$,

(ii) $\angle DOC$,

(iii) $\angle DAC$,

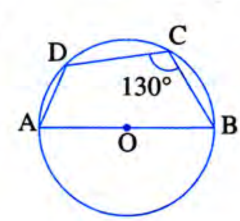
(iv) $\angle ADC$.



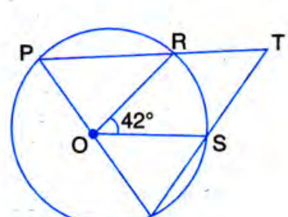
52. In the given figure, AB is the diameter of a circle with centre O. $\angle BCD = 130°$. Find :

(i) $\angle DAB$

(ii) $\angle DBA$



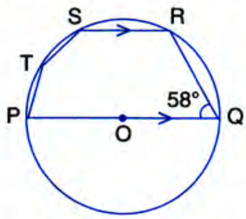
53. In the given figure, PQ is the diameter of the circle whose centre is O. Given $\angle ROS = 42°$, calculate $\angle RTS$.



54. In the given figure, PQ is a diameter. Chord SR is parallel to PQ. Given that ∠PQR = 58°, Calculate :

(i) ∠RPQ,

(ii) ∠STP.

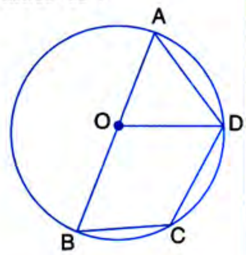


55. AB is the diameter of the circle with centre O. OD is parallel to BC and ∠AOD = 60°. Calculate the numerical values of :

(i) ∠ABD,

(ii) ∠DBC,

(iii) ∠ADC.



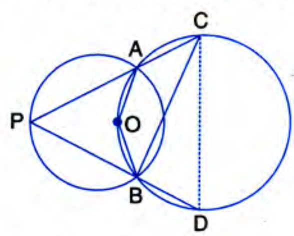
56. In the given figure, the centre O of the small circle lies on the circumference of the bigger circle. If ∠APB = 75° and ∠BCD = 40°, find :

(i) ∠AOB,

(ii) ∠ACB,

(iii) ∠ABD,

(iv) ∠ADB.

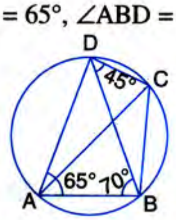


57. In the given figure, ∠BAD = 65°, ∠ABD = 70° and ∠BDC = 45°. Find :

(i) ∠BCD

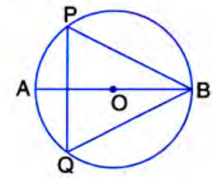
(ii) ∠ACB

Hence, show that AC is a diameter.



58. In a cyclic quadrilateral ABCD, ∠A : ∠C = 3 : 1 and ∠B : ∠D = 1 : 5; find each angle of the quadrilateral.

59. The given figure shows a circle with centre O and ∠ABP = 42°.



Calculate the measure of :

(i) ∠PQB

(ii) ∠QPB + ∠PBQ

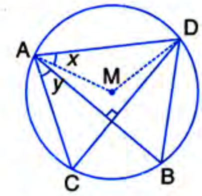
60. In the given figure, M is the centre of the circle. Chords AB and CD are perpendicular to each other.

If ∠MAD = x and ∠BAC = y :

(i) express ∠AMD in terms of x.

(ii) express ∠ABD in terms of y.

(iii) prove that : $x = y$.



61. In a circle, with centre O, a cyclic quadrilateral ABCD is drawn with AB as a diameter of the circle and CD equal to radius of the circle. If AD and BC produced meet at point P; show that ∠APB = 60°.

17.5 Some important results :

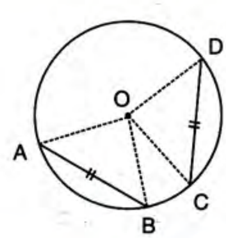
1. In a circle, equal chords subtend equal angles at the centre of the circle.

That is : if chord AB = chord CD

⇒ ∠AOB = ∠COD

Conversely, if ∠AOB = ∠COD

⇒ chord AB = CD

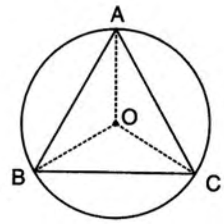


Clearly, O is the centre of the circle.

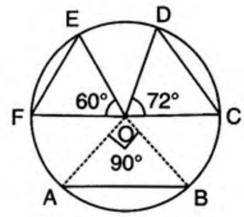
2. In the given figure, ABC is an equilateral triangle inscribed in a circle with centre O,

Since, AB = BC = AC

$\Rightarrow \angle AOB = \angle BOC = \angle AOC = \dfrac{360°}{3} = 120°$

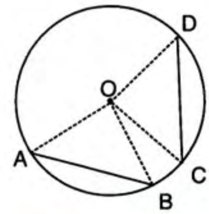


3. In the given figure, O is the centre of a circle. AB is the side of a square, CD is the side of a regular pentagon and EF is the side of a regular hexagon, then



$$\angle AOB = \frac{360°}{4} = 90°, \angle COD = \frac{360°}{5} = 72°$$

and $\angle EOF = \dfrac{360°}{6} = 60°$



4. In the given figure, if

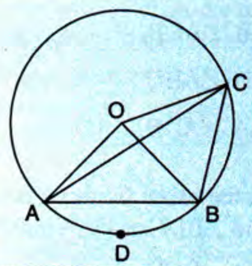
Chord AB : chord CD = 7 : 5,

$\angle AOB : \angle COD = 7 : 5$

And, if AB = 2CD $\Rightarrow \angle AOB = 2\angle COD$

14 In the given figure, the lengths of arc AB and arc BC are in the ratio 3 : 2. If angle $\angle AOB = 96°$; find :

(i) $\angle CAB$ (ii) $\angle ADB$



Solution :

Given : arc AB : arc BC = 3 : 2 $\Rightarrow \angle AOB : \angle BOC = 3 : 2$

$\Rightarrow 3\angle BOC = 2\angle AOB \Rightarrow \angle BOC = \frac{2}{3} \times 96° = 64°$

(i) Since, angle at the centre is twice the angle at the remaining circumference

$\Rightarrow \angle BOC = 2\angle CAB \Rightarrow$ **$\angle CAB = \dfrac{64°}{2} = 32°$** **Ans.**

(ii) $\angle AOB = 2\angle ACB \Rightarrow \angle ACB = \dfrac{96°}{2} = 48°$

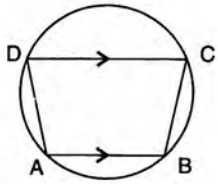
Now, ADBC is a cyclic quadrilateral $\Rightarrow \angle ADB + \angle ACB = 180°$

$\Rightarrow$ **$\angle ADB$** $= 180° - 48° =$ **$132°$** **Ans.**

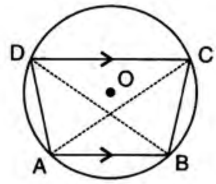
15 If two sides of a cyclic quadrilateral are parallel, prove that the other two sides are equal.

Solution :

Given : A cyclic quadrilateral ABCD incribed in a circle with centre O and sides AB and DC are parallel.



To Prove : AD = BC



Construction : Join AC and BD

Proof : Since, ABCD is a cyclic quadrilateral

$\angle ABC + \angle ADC = 180°$ ...I

[Opp. angles of a cyclic quadrilateral are supplementary]

Since, AB // DC and AD is transversal

$\therefore$ $\angle BAD + \angle ADC = 180°$ ...II [Co-interior angles]

From I and II, we get :

$\angle ABC + \angle ADC = \angle BAD + \angle ADC$

$\Rightarrow$ $\angle ABC = \angle BAD$

Now, in ΔBAD and ΔABC,

$\angle BAD = \angle ABC$ [Proved above]

$\angle ACB = \angle ADB$ [Angle of the same segment]

$\Rightarrow$ $\angle ABD = \angle BAC$ [When two angles of one Δ are equal to two angles of the other Δ, their third angles are also equal]

AB = AB [Common]

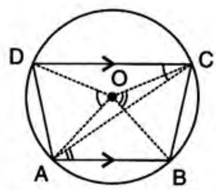
$\therefore$ $\Delta BAD \equiv \Delta ABC$ [By A.S.A.]

$\Rightarrow$ **AD = BC** [By C.P.C.T.C.]

Hence Proved.

Alternative method :

Join OA, OB, OC, OD and diagonal AC.



Since, angle at the centre is twice the angle at remaining circumference, therefore

$\angle BOC = 2\angle BAC$ I

and $\angle AOD = 2\angle ACD$ II

Since, AB // DC and AC is transversal

$\therefore$ $\angle BAC = \angle ACD$ ....III [Alternate angles]

From I, II and III, we get :

$\angle BOC = \angle AOD$

$\Rightarrow$ **AD = BC**

[Chords subtending equal angles at the centre are equal]

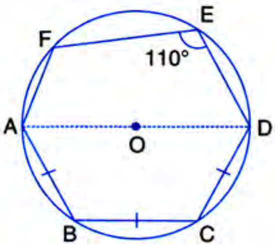
Hence Proved.

EXERCISE 17(B)

1. In a cyclic-trapezium, the non-parallel sides are equal and the diagonals are also equal. Prove it.

2. In the following figure, AD is the diameter of the circle with centre O. Chords AB, BC and CD are equal. If ∠DEF = 110°, calculate :

   (i) ∠AEF, (ii) ∠FAB.



3. If two sides of a cyclic-quadrilateral are parallel; prove that :

   (i) its other two sides are equal.

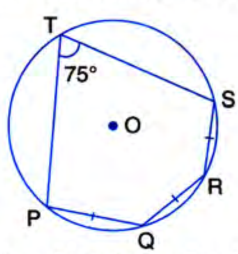
   (ii) its diagonals are equal.

4. The given figure shows a circle with centre O. Also, PQ = QR = RS and ∠PTS = 75°. Calculate :

   (i) ∠POS,

   (ii) ∠QOR,

   (iii) ∠PQR.

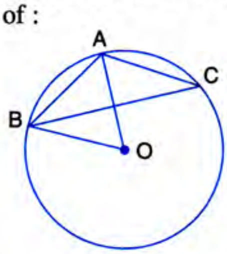


5. In the given figure, AB is a side of a regular six-sided polygon and AC is a side of a regular eight-sided polygon inscribed in the circle with centre O. Calculate the sizes of :

   (i) ∠AOB,

   (ii) ∠ACB,

   (iii) ∠ABC.



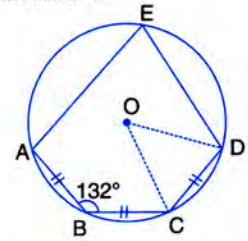
6. In a regular pentagon ABCDE, inscribed in a circle; find ratio between angle EDA and angle ADC.

7. In the given figure, AB = BC = CD and ∠ABC = 132°. Calculate :

   (i) ∠AEB,

   (ii) ∠AED,

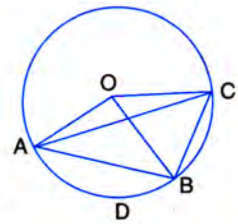
   (iii) ∠COD.



8. In the figure, O is the centre of the circle and the length of arc AB is twice the length of arc BC. If angle AOB = 108°, find :

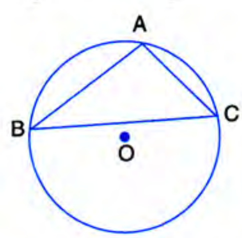
   (i) ∠CAB,

   (ii) ∠ADB.

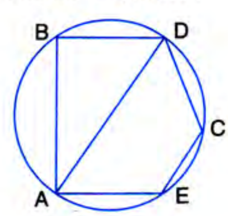


9. The figure shows a circle with centre O. AB is the side of regular pentagon and AC is the side of regular hexagon.

   Find the angles of triangle ABC.



10. In the given figure, BD is a side of a regular hexagon, DC is a side of a regular pentagon and AD is a diameter. Calculate :

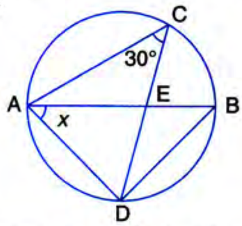


   (i) ∠ADC, (ii) ∠BDA,

   (iii) ∠ABC, (iv) ∠AEC.

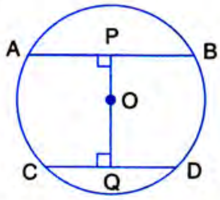
EXERCISE 17(C)

1. In the given circle with diameter AB, find the value of x.

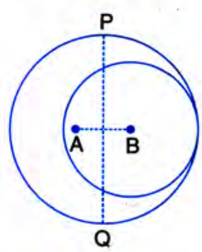


[2003]

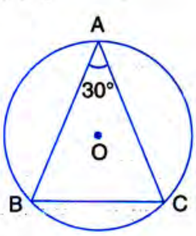
2. In the given figure, O is the centre of the circle with radius 5 cm. OP and OQ are perpendiculars to AB and CD respectively. AB = 8 cm and CD = 6 cm. Determine the length of PQ.



3. The given figure shows two circles with centres A and B; and radii 5 cm and 3 cm respectively, touching each other internally. If the perpendicular bisector of AB meets the bigger circle in P and Q, find the length of PQ.

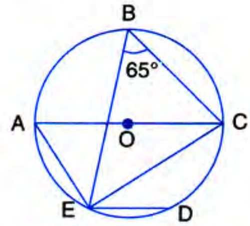


4. In the given figure, ABC is a triangle in which ∠BAC = 30°. Show that BC is equal to the radius of the circumcircle of the triangle ABC, whose centre is O.



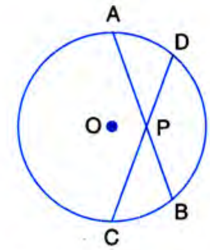
5. Prove that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.

6. In the given figure, chord ED is parallel to diameter AC of the circle. Given ∠CBE = 65°, calculate ∠DEC.



7. Chords AB and CD of a circle intersect each other at point P such that AP = CP.

Show that : AB = CD.



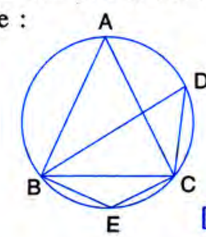
8. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic. Prove it.

9. In the figure, ∠DBC = 58°. BD is a diameter of the circle. Calculate :

(i) ∠BDC

(ii) ∠BEC

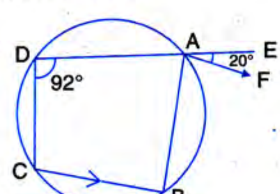
(iii) ∠BAC



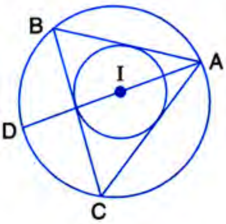
[2014]

10. D and E are points on equal sides AB and AC of an isosceles triangle ABC such that AD = AE. Prove that the points B, C, E and D are concyclic.

11. In the given figure, ABCD is a cyclic quadrilateral. AF is drawn parallel to CB and DA is produced to point E. If ∠ADC = 92°, ∠FAE = 20°; determine ∠BCD. Give reason in support of your answer.



12. If I is the incentre of triangle ABC and AI when produced meets the circumcircle of triangle ABC in point D. If ∠BAC = 66° and ∠ABC = 80°. Calculate:



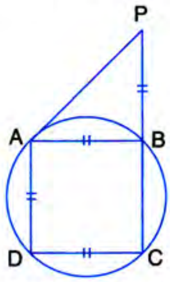
(i) ∠DBC, (ii) ∠IBC,

(iii) ∠BIC.

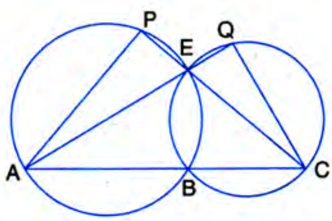
13. In the given figure, AB = AD = DC = PB and ∠DBC = x°. Determine, in terms of x :

(i) ∠ABD, (ii) ∠APB.

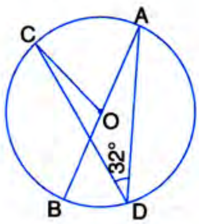
Hence or otherwise, prove that AP is parallel to DB.



14. In the given figure; ABC, AEQ and CEP are straight lines. Show that ∠APE and ∠CQE are supplementary.



15. In the given figure, AB is the diameter of the circle with centre O.

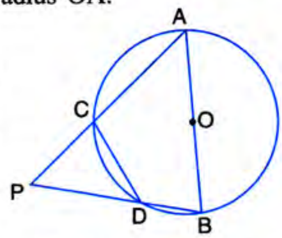


If ∠ADC = 32°, find angle BOC.

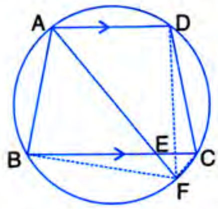
16. In a cyclic-quadrilateral PQRS, angle PQR = 135°. Sides SP and RQ produced meet at point A : whereas sides PQ and SR produced meet at point B.
If ∠A : ∠B = 2 : 1; find angles A and B.

17. In the following figure, AB is the diameter of a circle with centre O and CD is the chord with length equal to radius OA.

If AC produced and BD produced meet at point P; show that : ∠APB = 60°.



18. In the following figure, ABCD is a cyclic quadrilateral in which AD is parallel to BC.



If the bisector of angle A meets BC at point E and the given circle at point F, prove that :

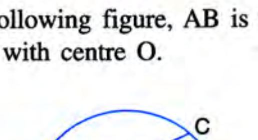
(i) EF = FC (ii) BF = DF

19. ABCD is a cyclic quadrilateral. Sides AB and DC produced meet at point E; whereas sides BC and AD produced meet at point F.

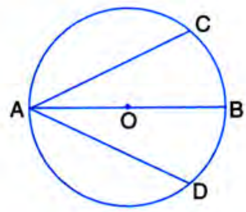
If ∠DCF : ∠F : ∠E = 3 : 5 : 4, find the angles of the cyclic quadrilateral ABCD.

20. The following figure shows a circle with PR as its diameter.

If PQ = 7 cm and QR = 3RS = 6 cm, find the perimeter of the cyclic quadrilateral PQRS.



21. In the following figure, AB is the diameter of a circle with centre O.



If chord AC = chord AD, prove that :

(i) arc BC = arc DB

(ii) AB is bisector of ∠CAD.

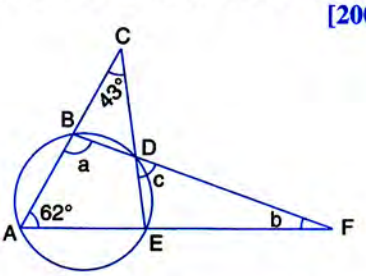
Further, if the length of arc AC is twice the length of arc BC, find : (a) ∠BAC (b) ∠ABC

22. In cyclic quadrilateral ABCD; AD = BC, ∠BAC = 30° and ∠CBD = 70°; find :

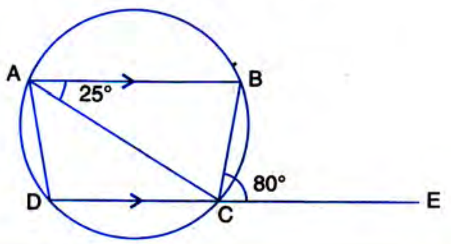
(i) ∠BCD (ii) ∠BCA

(iii) ∠ABC (iv) ∠ADC

23. In the given figure, ∠ACE = 43° and ∠CAF = 62°; find the values of *a*, *b* and *c*. **[2007]**



24. In the given figure, AB is parallel to DC, ∠BCE = 80° and ∠BAC = 25°.



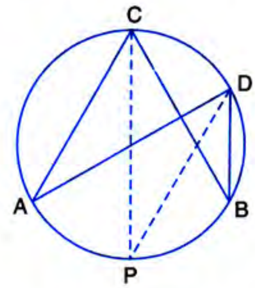
Find :

(i) ∠CAD (ii) ∠CBD (iii) ∠ADC

25. ABCD is a cyclic quadrilateral of a circle with centre O such that AB is a diameter of this circle and the length of the chord CD is equal to the radius of the circle. If AD and BC produced meet at P, show that APB = 60°.

26. In the figure, given alongside, CP bisects angle ACB.

Show that DP bisects angle ADB.



27. In the figure, given below, AD = BC, ∠BAC = 30° and ∠CBD = 70°.

Find :

(i) ∠BCD

(ii) ∠BCA

(iii) ∠ABC

(iv) ∠ADB