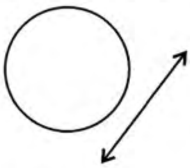
18 Tangents and Intersecting Chords

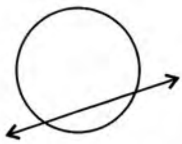
18.1 Introduction :

If a circle and a straight line are drawn in a plane; then with respect to each other, they may have one of the following three positions :

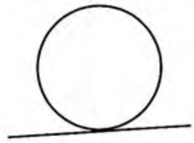
First position : The line does not meet (cut) the circle.



Second position : The line cuts the circle at two points.



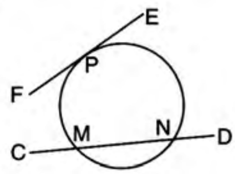
Third position : The line meets (touches) the circle at one point only.



The straight line which cuts a circle at two points is called the ***secant of the circle***.

In the given figure, CD is a secant of the given circle.

The line which touches a circle at one point only is called the ***tangent of the circle***.



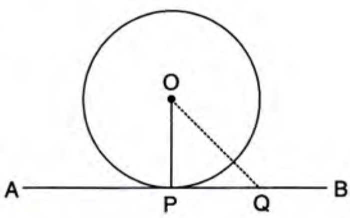
In the given figure, EF is tangent to the given circle at point P of the circle.

The point at which a tangent touches the circle is called the ***point of contact***.

Theorem 10

The tangent at any point of a circle and the radius through this point are perpendicular to each other.

Given : A circle with centre O. AB is a tangent to the circle at point P and OP is the radius of the circle.



To Prove : OP $\perp$ AB.

Construction : Take a point Q (other than P) on the tangent AB. Join OQ.

> ***Remember :***
> Out of all the line segments drawn from a given point to a given line; the perpendicular is the shortest.

Proof :

Statement	*Reason*
1. OP < OQ	Since, each point of the tangent, other than point P, is outside the circle.

2. Similarly, it can be shown that out of all the line-segments which would be drawn from point O to the tangent line AB; OP is the shortest.

3. ∴ **OP ⊥ AB** — The shortest line segment, drawn from a given point to a given line, is perpendicular to the line.

Hence Proved.

> ***Remember :***
> 1. No tangent can be drawn to a circle through a point inside the circle.
> 2. One and only one tangent can be drawn through a point on the circumference of the circle.
> 3. Only two tangents can be drawn to a circle through a point outside the circle.

Corollary : If two tangents are drawn to a circle from an exterior point (the point which lies outside the circle) :

(i) the tangents are equal in length;

(ii) the tangents subtend equal angles at the centre of the circle and

(iii) the tangents are equally inclined to the line joining the point and the centre of the circle.

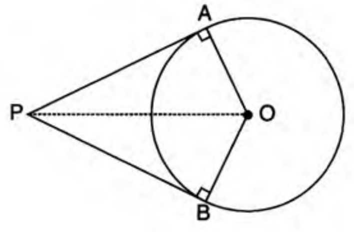
Given : A circle with centre O. PA and PB are two tangents drawn to this circle, from an exterior point P.

To Prove :

(i) PA = PB,

(ii) ∠AOP = ∠BOP,

(iii) ∠APO = ∠BPO.



Proof :

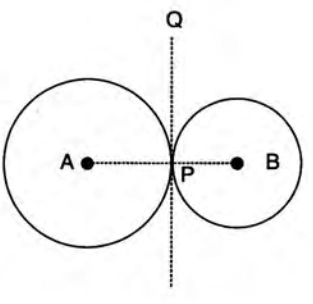
Statement	***Reason***
In Δ AOP and Δ BOP;	
OA = OB	Radii of the same circle
∠OAP = ∠OBP = 90°	Angle between the radius and the tangent is 90°
OP = OP	Common
∴ Δ AOP ≅ Δ BOP	by R.H.S.
∴ (i) **PA = PB**	Corresponding parts of congruent triangles are congruent.
(ii) **∠AOP = ∠BOP**	
(iii) **∠APO = ∠BPO**	

Hence Proved.

Theorem 11

If two circles touch each other, the point of contact lies on the straight line through the centres.

Case I : *When the given two circles touch each other externally.*



Given : Two circles with centres A and B touching each other externally at point P.

To Prove : P lies on the line AB.

Construction : Through the point of contact P, draw a common tangent PQ. Join AP and BP.

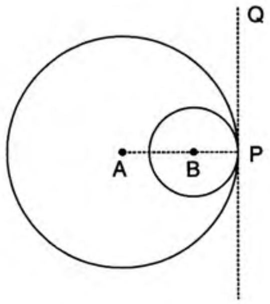
Proof :

Statement	***Reason***
1. $\angle APQ = 90°$	Angle between radius and tangent
2. $\angle BPQ = 90°$	Angle between radius and tangent
3. $\angle APQ + \angle BPQ = 180°$	Adding (1) and (2).
$\Rightarrow$ $\angle APB = 180°$	
$\Rightarrow$ APB is a straight line	Straight line angle = 180°

$\therefore$ **P lies on the line AB.**

Hence Proved.

Case II : *When the given two circles touch each other internally.*



Given : Two circles with centres A and B touching each other internally at point P.

To Prove : P lies on the line AB produced.

Construction : Through the point of contact P, draw a common tangent PQ. Join AP and BP.

Proof :

Statement	***Reason***
1. $\angle APQ = 90°$	Angle between the radius and the tangent
2. $\angle BPQ = 90°$	Angle between the radius and the tangent
$\therefore$ AP and BP both are perpendicular to the tangent PQ at the same point P.	From (1) and (2)
3. AP and BP lie in the same line.	Only one perpendicular can be drawn to a line through a point in it.

$\Rightarrow$ ABP is a straight line.

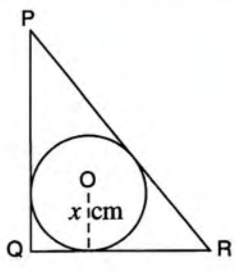
$\therefore$ **P lies on the line AB (when produced).**

Hence Proved.

Remember :

1. If r_1 and r_2 be radii of two circles touching each other at a point and d be the distance between their centres then :
   (i) $d = r_1 + r_2$ when circles touch each other *externally*,
   (ii) $d = r_1 \sim r_2$ when circles touch each other *internally*,
   i.e. $d = r_1 - r_2$ when r_1 is greater and $d = r_2 - r_1$ when r_2 is greater.
2. If AB and CD are tangents to the same circle at points P and Q such that AB is parallel to CD, then PQ is always the diameter of that circle.
3. Concentric circles means circles with the same centre.

1 In triangle PQR, PQ = 24 cm, QR = 7 cm and $\angle PQR = 90°$. Find the radius of the inscribed circle. [2012]



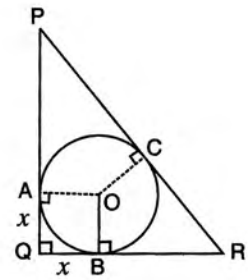
Solution :

In ΔPQR, angle PQR = 90°

$\Rightarrow PR^2 = PQ^2 + QR^2$

$= (24 \text{ cm})^2 + (7 \text{ cm})^2$ *i.e.* PR = 25 cm

Let the given inscribed circle touches the sides of the given triangle at points A, B and C respectively as shown. Clearly, OAQB is a square and so



AQ = BQ = x cm,

PA = PQ – AQ = $(24 - x)$ cm and

RB = QR – BQ = $(7 - x)$ cm

As tangents to a circle, from an exterior point are equal, PC = PA = $(24 - x)$ cm

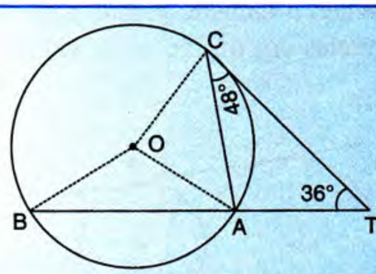
and RC = RB = $(7 - x)$ cm

PR = PC + RC $\Rightarrow 25 = (24 - x) + (7 - x)$

i.e. $25 = 31 - 2x$ *i.e.* $2x = 6$ and $x = 3$

$\Rightarrow$ **Radius of the inscribed circle = 3 cm.** **Ans.**

2 A, B and C are three points on a circle. The tangent at C meets BA produced at T. Given that $\angle ATC = 36°$ and that $\angle ACT = 48°$, calculate the angle subtended by AB at the centre of the circle. **[2001]**



Solution :

Mark O, the centre of the circle. Join OA, OB and OC.

Since, the angle between the radius and the tangent is 90° $\Rightarrow \angle OCT = 90°$

$\therefore \angle OCA = 90° - 48° = 42°$

And, OA = OC $\Rightarrow \angle OAC = \angle OCA = 42°$

In Δ ACT, exterior angle CAB = 48° + 36° = 84°

$\therefore \angle OAB = \angle CAB - \angle OAC = 84° - 42° = 42°$

Also, OA = OB $\Rightarrow \angle OBA = \angle OAB = 42°$

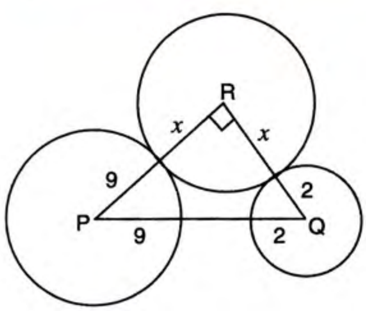
$\therefore$ In Δ OBA, $\angle BOA = 180° - \angle OAB - \angle OBA$

$= 180° - 42° - 42° = 96°$ **Ans.**

3 P and Q are centres of circles with radii 9 cm and 2 cm respectively. PQ = 17 cm. R is the centre of a circle of radius x cm, which touches the above circles externally. Given that $\angle PRQ = 90°$, write an equation in x and solve it. **[2004]**

Solution :

According to the given statement, the figure will be as shown alongside, in which PQ = 17 cm and angle PRQ = 90°.



We know when two circles touch each other externally, the distance between their centres is equal to the sum of their radii.

$\therefore$ PR = $(9 + x)$ cm and QR = $(2 + x)$ cm.

Applying Pythagoras Theorem, we get :

$PR^2 + QR^2 = PQ^2 \Rightarrow (9 + x)^2 + (2 + x)^2 = 17^2$

i.e. $81 + 18x + x^2 + 4 + 4x + x^2 = 289$

$\Rightarrow 2x^2 + 22x - 204 = 0$ *i.e.* $x^2 + 11x - 102 = 0$

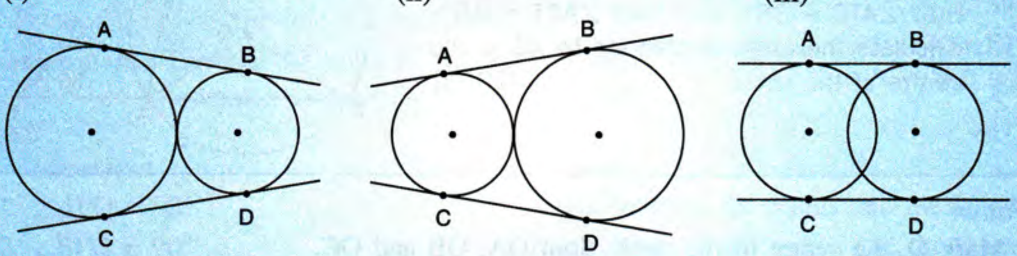
$\Rightarrow x^2 + 17x - 6x - 102 = 0$ *i.e.* $(x + 17)(x - 6) = 0$

$\Rightarrow x = -17$ or $x = 6$ *i.e.* $\mathbf{x = 6}$ **Ans.**

4 Two circles with radii 25 cm and 9 cm touch each other externally. Find the length of the direct common tangent.

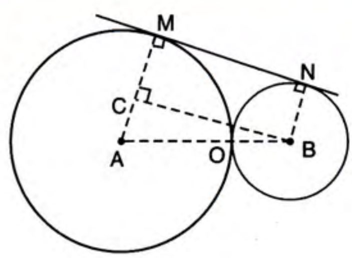
Solution :

When a tangent, common to two given circles, is drawn in such a way that both the circles are on the same side of it, the tangent is called a **direct common tangent**.



In each of the figures, given above, AB and CD are two direct common tangents which are always equal in length *i.e.* in each case, AB = CD.

According to the given statement, the figure will be as shown alongside :



In the figure, A and B are the centres of the two circles touching each other externally at point O. Also, MN is the direct common tangent.

Draw $BC \perp AM$.

Clearly, BCMN is a rectangle and so BC = MN.

Now, AB = OA + OB = 25 cm + 9 cm = 34 cm

AC = AM – BN = 25 cm – 9 cm = 16 cm

In right-angled triangle ABC

$AB^2 = AC^2 + BC^2 \Rightarrow 34^2 = 16^2 + BC^2$

$\Rightarrow BC^2 = 1156 - 256 = 900$

$\Rightarrow BC = 30$

$\therefore$ **Length of direct common tangent** = MN = BC

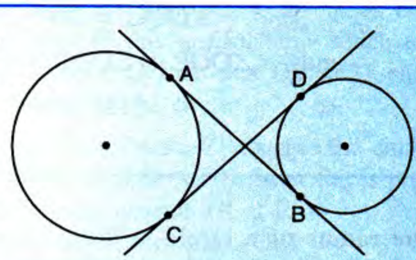
= **30 cm** **Ans.**

5 The centres of two circles with radii 6 cm and 2 cm are 10 cm apart. Calculate the length of the transverse common tangent.

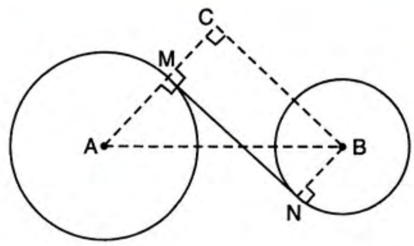
Solution :

When a tangent, common to two given circles, is drawn in such a way that both the circles are on either side of it, the tangent is called a **transverse common tangent**.

In the figure given alongside, AB and CD are two transverse common tangents which are always equal *i.e.* AB = CD.



According to the given statement, the figure will be as shown alongside :



In the figure, A and B are centres of the two circles with radii 6 cm and 2 cm. Also, MN is the transverse common tangent.

Hence, AB = 10 cm, AM = 6 cm and BN = 2 cm. Draw BC perpendicular to AM produced.

Clearly, BCMN is a rectangle, MN = BC and CM = BN = 2 cm.

Now, AC = AM + CM = 6 cm + 2 cm = 8 cm

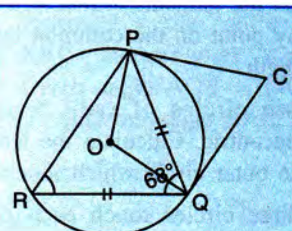
In right-angled triangle ABC,

$$AB^2 = BC^2 + AC^2 \quad \Rightarrow \quad BC^2 = AB^2 - AC^2$$
$$= 10^2 - 8^2 = 100 - 64 = 36$$
$$\Rightarrow \quad BC = 6$$

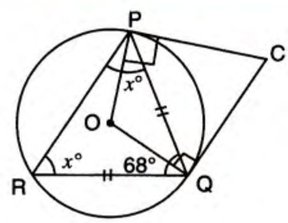
∴ **Length of transverse common tangent** = MN = BC

= **6 cm** **Ans.**

6 In the figure, given alongside, PQ = QR, ∠RQP = 68°, PC and QC are tangents to the circle with centre O. Calculate the values of (i) ∠QOP (ii) ∠QCP.



Solution :



(i) In Δ PQR, PQ = QR

$\Rightarrow$ ∠QPR = ∠QRP = x (let)

∴ $x + x + 68° = 180°$

$\Rightarrow$ $x = 56°$

i.e. ∠QRP = 56°

Since, angle at centre is twice the angle at remaining circumference, therefore

∠QOP = 2∠QRP = 2 × 56°

= **112°** **Ans.**

(ii) In quadrilateral POQC,

∠QOP = 112° [Proved above]

$\angle OPC = 90°$ [Angle between radius and tangent]

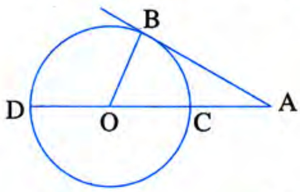
$\angle OQC = 90°$ [Angle between radius and tangent]

and $\angle QOP + \angle OPC + \angle OQC + \angle QCP = 360°$

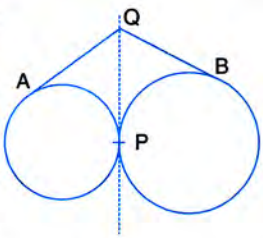
$\Rightarrow$ $112° + 90° + 90° + \angle QCP = 360°$ $\Rightarrow$ **$\angle QCP = 68°$** **Ans.**

EXERCISE 18(A)

1. The radius of a circle is 8 cm. Calculate the length of a tangent drawn to this circle from a point at a distance of 10 cm from its centre.
2. In the given figure, O is the centre of the circle and AB is a tangent at B. If AB = 15 cm and AC = 7·5 cm, calculate the radius of the circle. **[2012]**

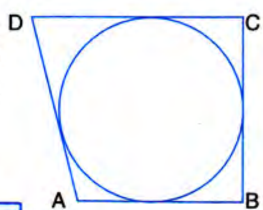


3. Two circles touch each other externally at point P. Q is a point on the common tangent through P. Prove that the tangents QA and QB are equal.



4. Two circles touch each other internally. Show that the tangents drawn to the two circles from any point on the common tangent, are equal in length.
5. Two circles of radii 5 cm and 3 cm are concentric. Calculate the length of a chord of the outer circle which touches the inner.
6. Three circles touch each other externally. A triangle is formed when the centres of these circles are joined together. Find the radii of the circles, if the sides of the triangle formed are 6 cm, 8 cm and 9 cm.
7. If the sides of a quadrilateral ABCD touch a circle, prove that :

   AB + CD = BC + AD.



Let AB, BC, CD and DA touch the circle at points P, Q, R and S respectively. Since, tangents to a circle from an exterior point are equal in length, therefore,

AP = AS, BP = BQ, DR = DS and CR = CQ.

On adding, we get :

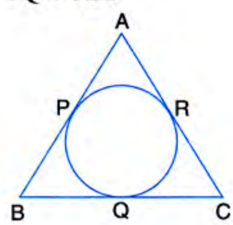
AP + BP + DR + CR

= AS + BQ + DS + CQ

$\Rightarrow$ AB + CD = AD + BC.

8. If the sides of a parallelogram touch a circle (refer figure of Q. 7), prove that the parallelogram is a rhombus.
9. From the given figure, prove that :

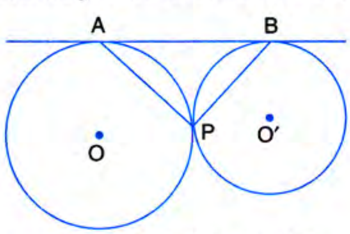
   AP + BQ + CR = BP + CQ + AR.



   Also, show that :

   AP + BQ + CR = $\frac{1}{2}$ × Perimeter of Δ ABC.

10. In the figure of Q. 9; if AB = AC then prove that : BQ = CQ.
11. Radii of two circles are 6·3 cm and 3·6 cm. State the distance between their centres if :
    (i) they touch each other externally,
    (ii) they touch each other internally.
12. From a point P outside a circle, with centre O, tangents PA and PB are drawn. Prove that :
    (i) $\angle AOP = \angle BOP$,
    (ii) OP is the ⊥ bisector of chord AB.
13. In the given figure, two circles touch each other externally at point P. AB is the direct common tangent of these circles. Prove that :



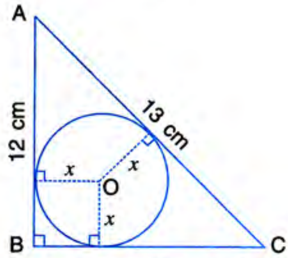
    (i) tangent at point P bisects AB,
    (ii) angle APB = 90°.

14. Tangents AP and AQ are drawn to a circle, with centre O, from an exterior point A. Prove that :

$$\angle PAQ = 2\angle OPQ$$

15. Two parallel tangents of a circle meet a third tangent at points P and Q. Prove that PQ subtends a right angle at the centre.

16. ABC is a right angled triangle with AB = 12 cm and AC = 13 cm. A circle, with centre O, has been inscribed inside the triangle.



Calculate the value of x, the radius of the inscribed circle. **[2005 type]**

17. In a triangle ABC, the incircle (centre O) touches BC, CA and AB at points P, Q and R respectively. Calculate :

(i) $\angle QOR$ (ii) $\angle QPR$;

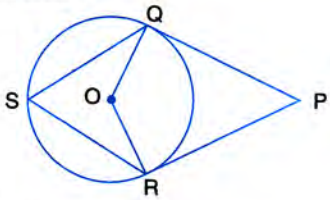
given that $\angle A = 60°$.

18. In the following figure, PQ and PR are tangents to the circle, with centre O. If $\angle QPR = 60°$, calculate :

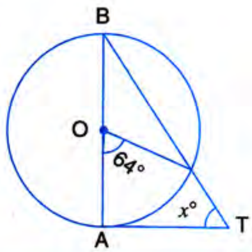
(i) $\angle QOR$,

(ii) $\angle OQR$,

(iii) $\angle QSR$.



19. In the given figure, AB is the diameter of the circle, with centre O, and AT is the tangent. Calculate the numerical value of x.

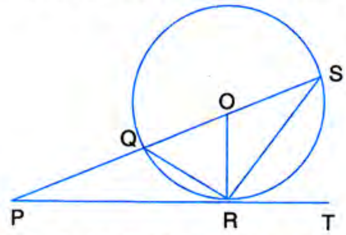


20. In quadrilateral ABCD; angle D = 90°, BC = 38 cm and DC = 25 cm. A circle is inscribed in this quadrilateral which touches AB at point Q such that QB = 27 cm. Find the radius of the circle.

21. In the given figure, PT touches the circle with centre O at point R. Diameter SQ is produced to meet the tangent TR at P.
Given $\angle SPR = x°$ and $\angle QRP = y°$;
prove that :

(i) $\angle ORS = y°$

(ii) write an expression connecting x and y.

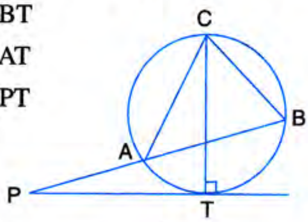


22. PT is a tangent to the circle at T. If $\angle ABC = 70°$ and $\angle ACB = 50°$; calculate :

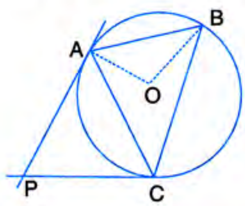
(i) $\angle CBT$

(ii) $\angle BAT$

(iii) $\angle APT$

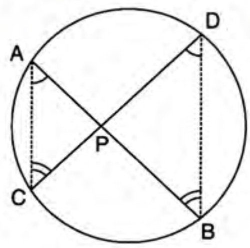


23. In the given figure, O is the centre of the circumcircle ABC. Tangents at A and C intersect at P. Given angle AOB = 140° and angle APC = 80°; find the angle BAC.



Theorem 12

If two chords of a circle intersect internally or externally then the product of the lengths of their segments is equal.



Case I : *When chords intersect internally*

Given : Chords AB and CD of a circle intersect each other at point P inside the circle

To Prove : PA × PB = PC × PD

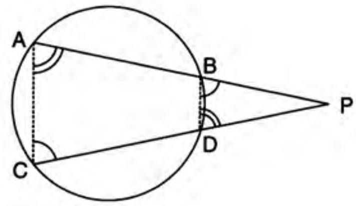
Construction : Join AC and BD.

Proof :

Statement	*Reason*
In Δ APC and Δ BPD,	
$\angle A = \angle D$	Angles of the same segment
$\angle C = \angle B$	"
⇒ Δ APC ~ Δ BPD	By A.A. Postulate
⇒ $\frac{PA}{PD} = \frac{PC}{PB}$	Corresponding sides of similar Δs are proportional
⇒ **PA × PB = PC × PD**	

Hence Proved.

Case II : *When chords intersect externally.*



Given : Chords AB and CD of a circle, when produced, intersect each other at point P outside the circle.

To Prove : PA × PB = PC × PD

Construction : Join AC and BD.

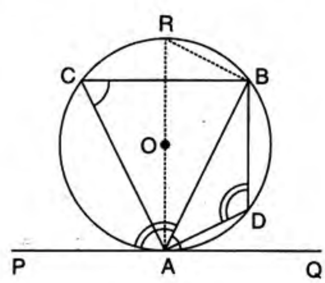
Proof :

Statement	*Reason*
In Δ PAC and Δ PDB,	
$\angle A = \angle PDB$	Ext. angle of a cyclic quad. = Int. opp. angle
$\angle C = \angle PBD$	" "
⇒ Δ PAC ~ Δ PDB	By A.A. Postulate
⇒ $\frac{PA}{PD} = \frac{PC}{PB}$	Corresponding sides of similar triangles are proportional
⇒ **PA × PB = PC × PD**	

Hence Proved.

Theorem 13

The angle between a tangent and a chord through the point of contact is equal to an angle in the alternate segment.



Given : A circle with centre O. Tangent PQ touches the circle at point A. Through A, the point of contact, a chord AB is drawn.

To Prove : The angle between tangent PQ and chord AB through the point of contact A is equal to the angle in the alternate segment *i.e.* if C is point on major arc AB and D is a point on minor arc AB; then

$$\angle BAQ = \angle ACB$$

$$\text{and, } \angle BAP = \angle ADB$$

Construction : Draw the diameter AOR and join RB.

Proof :

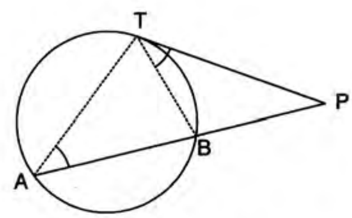
Statement	**Reason**
In Δ ABR,	
$\angle ABR = 90°$	Angle of a semi-circle
$\Rightarrow \angle ARB + \angle RAB = 90°$ I	
$\angle OAQ = 90°$	Angle between the radius and the tangent
$\Rightarrow \angle RAB + \angle BAQ = 90°$ II	
$\therefore \angle ARB + \angle RAB = \angle RAB + \angle BAQ$	From I & II
$\Rightarrow \angle ARB = \angle BAQ$ III	
But, $\angle ARB = \angle ACB$ IV	Angles of the same segment
$\therefore$ $\boldsymbol{\angle BAQ = \angle ACB}$	From III & IV

Hence Proved

Now, $\angle BAP + \angle BAQ = 180°$	Straight line angles
and $\angle ACB + \angle ADB = 180°$	Opp. angles of cyclic quad.
$\Rightarrow \angle BAP + \angle BAQ = \angle ACB + \angle ADB$	
$\Rightarrow$ $\boldsymbol{\angle BAP = \angle ADB}$	As, $\angle BAQ = \angle ACB$

Hence Proved.

Theorem 14



If a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.

Given : Chord AB and tangent TP of a circle intersect each other at point P outside the circle.

To Prove : $PA \times PB = PT^2$

Construction : Join TA and TB.

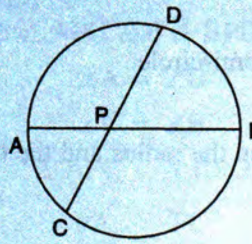
Proof :

Statement	***Reason***
In Δ PAT and Δ PTB,	
$\angle PTB = \angle A$	Angle in the alternate segment
$\angle P = \angle P$	Common
$\Rightarrow \Delta PAT \sim \Delta PTB$	By A.A. Postulate
$\Rightarrow \dfrac{PA}{PT} = \dfrac{PT}{PB}$	Corresponding sides of similar Δs are proportional
$\Rightarrow$ $\boldsymbol{PA \times PB = PT^2}$	

Hence Proved.

7 From each of the following figures, find the value of x.

(i)

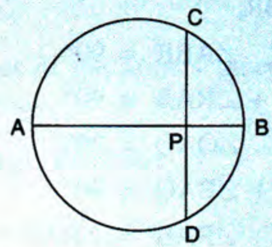


PA = 4 cm, PB = 6 cm,

PC = 5 cm and PD = x cm

and

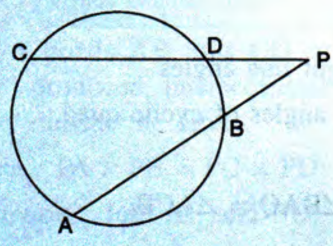
(ii)



PA = 2PB = 12 cm

PC = PD = x cm

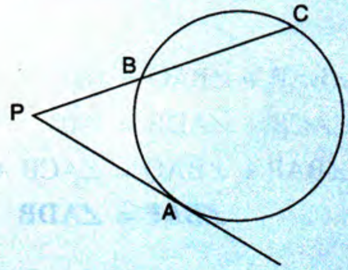
(iii)



AB = 10 cm, PB = 6 cm,

CD = x cm and PD = 4 cm

(iv)



PA = 20 cm, PB = 16 cm

and BC = x cm.

Solution :

(i) Since, chords AB and CD intersect each other at point P, inside the circle :

$\Rightarrow \quad PA \times PB = PC \times PD$

$\Rightarrow \quad 4\text{ cm} \times 6\text{ cm} = 5\text{ cm} \times x\text{ cm}$

$\Rightarrow \quad x = \frac{24}{5}\text{ cm} = \mathbf{4.8\text{ cm}}$ **Ans.**

(ii) Given, PA = 12 cm, PB = $\frac{12}{2}$ cm = 6 cm; chords AB and CD intersect each other at point P, inside the circle :

$\Rightarrow \quad PA \times PB = PC \times PD$

$\Rightarrow \quad 12\text{ cm} \times 6\text{ cm} = x\text{ cm} \times x\text{ cm}$

$\Rightarrow \quad x^2 = 72$

$\Rightarrow \quad x = \sqrt{72}\text{ cm} = \mathbf{6\sqrt{2}\text{ cm}}$ **Ans.**

(iii) Since, chords AB and CD intersect each other, externally at point P

$\Rightarrow \quad PA \times PB = PC \times PD$

$\Rightarrow \quad 16 \times 6 = (4 + x) \times 4$ | PA = AB + PB = 10 + 6 = 16 cm; PC = PD + CD = $(4 + x)$ cm

$x = \mathbf{20\text{ cm}}$ **Ans.**

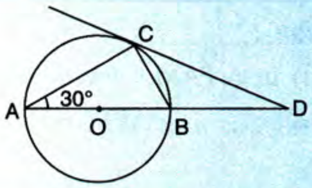
(iv) Since, chord BC and the tangent at point A intersect each other at point P, outside the circle :

$$\Rightarrow \quad PA^2 = PB \times PC$$

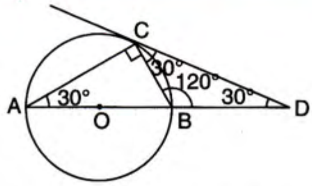
$$\Rightarrow \quad (20)^2 = 16 \times (16 + x) \qquad [PC = PB + BC = (16 + x) \text{ cm}]$$

$$x = 9 \text{ cm}$$ **Ans.**

8 In the given figure, AB is the diameter and AC is the chord of a circle such that ∠BAC = 30°. The tangent at C intersects AB produced at D. Prove that : BC = BD. **[2004]**



Solution :



∠BAC = 30°

⇒ ∠BCD = 30° [Angle of alt. segment]

∠ACB = 90° [Angle of semi-circle]

In Δ ACB,

∠CBD = 30° + 90° = 120° [Ext. ∠ = Sum of int. opp. ∠s]

In Δ BCD,

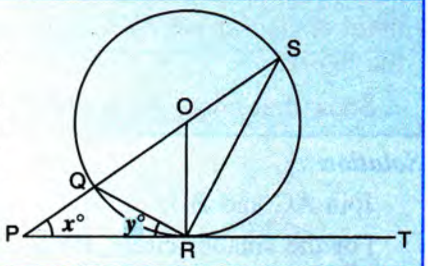
∠BDC = 180° – (120° + 30°) = 30°

Since, ∠BCD = ∠BDC [Each 30°]

∴ **BC = BD** [Sides opp. to equal angles]

Hence Proved.

9 In the given figure, PT touches a circle with centre O at R. Diameter SQ when produced meets PT at P. If ∠SPR = x° and ∠QRP = y°, show that $x° + 2y° = 90°$. **[2006]**



Solution :

∠RSQ = $y°$ [Angle in alt. segment]

In Δ PQR, ∠RQS = $x° + y°$ [Ext. ∠ of a Δ = Sum of its int. opp. ∠s]

In Δ QRS, ∠QRS = 90° [Angle of semi-circle]

⇒ ∠RSQ + ∠RQS = 90°

⇒ $y° + (x° + y°) = 90°$

⇒ $x° + 2y° = 90°$ **Hence the required result.**

Alternative method :

∠S = $y°$ [Angle in alt. segment]

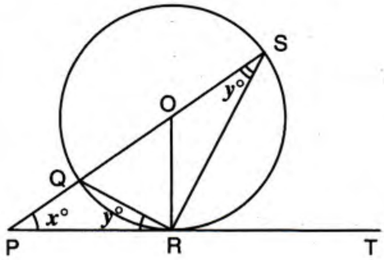
∠QRS = 90° [Angle of semi-circle]

In ΔPRS, ∠P + ∠PRS + ∠S = 180° [?]

⇒ $x° + (90 + y°) + y° = 180°$

⇒ $x° + 2y° = 90°$

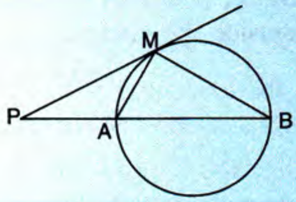
Hence the required result.



10 In the given figure, PM is a tangent to the circle and PA = AM. Prove that :

(i) Δ PMB is isosceles

(ii) PA × PB = MB^2 [2005]

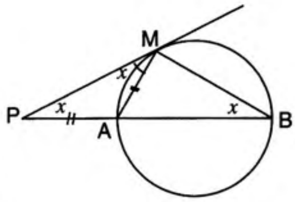


Solution :

(i) In Δ PAM, PA = AM

⇒ ∠APM = ∠AMP = x (suppose)

[∠s opp. to equal sides]



Also, ∠ABM = ∠AMP = x

[Angle of alternate segment]

⇒ ∠APM = ∠ABM = x ⇒ MB = MP

∴ **Δ PMB is isosceles.** **Hence Proved.**

(ii) Since the chord AB produced and the tangent at point M intersect each other externally at point P.

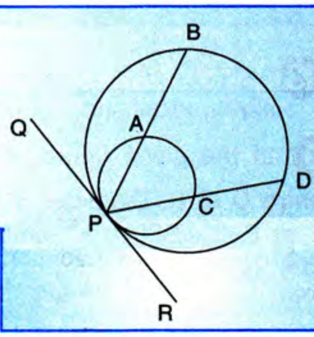
∴ PA × PB = MP^2

⇒ **PA × PB = MB^2** [∵ MB = MP, proved above]

Hence Proved.

11 Two circles touch each other internally at point P. QPR is the tangent at P; segments PAB and PCD meet circles at points A, B, C and D as shown in the figure.

Show that chord AC is parallel to chord BD.



Solution :

Join AC and BD.

For the smaller circle, PA is the chord and QPR is the tangent at point P

⇒ ∠QPA = ∠ACP .....I [Angle of alternate segment]

For the bigger circle, PB is the chord and QPR is the tangent at point P

⇒ ∠QPB = ∠BDP .....II [Angle of alternate segment]

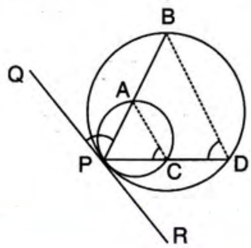
From I and II, we get :

∠ACP = ∠BDP | As, ∠QPA = ∠QPB |

But these are corresponding angles and whenever the corresponding angles are equal, the lines are parallel.

⇒ **AC is parallel to BD.**

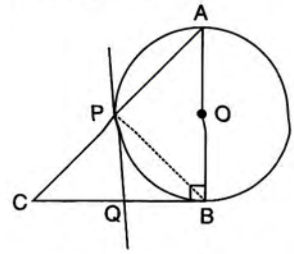
Hence Proved.



12 In a right triangle ABC, a circle with AB as diameter is drawn to intersect the hypotenuse AC in P. Prove that the tangent at P, bisects the side BC.

Solution :

According to the given statement, the figure will be as drawn alongside.



Let the tangent at point P to the circle with AB diameter meets side BC at point Q.

To Prove : CQ = BQ

Construction : Join PB

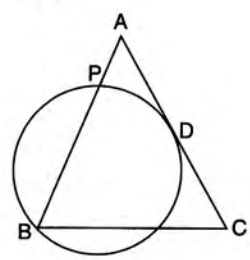
Proof : Since, angle of semi-circle is 90°

$\Rightarrow$	$\angle APB = 90°$			
$\Rightarrow$	$\angle BPC = 90°$			[APC is a straight line]
$\Rightarrow$	$\angle BPQ + \angle CPQ = 90°$	I		
In Δ ABC,	$\angle ABC = 90°$			[Given]
$\Rightarrow$	$\angle A + \angle C = 90°$	II		
Also,	$\angle BPQ = \angle A$	III		[Angle in the alternate segment]
$\Rightarrow$	$\angle A + \angle CPQ = 90°$	IV		[From I and III]
$\Rightarrow$	$\angle A + \angle CPQ = \angle A + \angle C$			[From II to IV]
$\Rightarrow$	$\angle CPQ = \angle C$			
$\Rightarrow$	PQ = CQ			
But,	PQ = BQ			[Tangents to the circle from the ext. point Q]
$\therefore$	**CQ = BQ**			**Hence Proved.**

13 ABC is an isosceles triangle with AB = AC. A circle through B touches side AC at its middle point D and intersects side AB in point P. Show that : AB = 4 × AP.

Solution :

According to the given statement, the figure will be as drawn below :



Since, AD is tangent to the circle and BP is its chord intersecting the tangent AD at point A.

$\therefore \quad AD^2 = AP \times AB$

Since, D is mid-point of AC

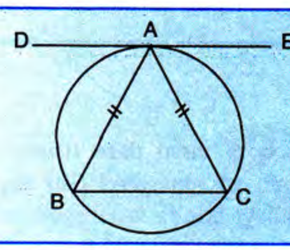
$\therefore \quad AD = \quad AC = \frac{1}{2}AB$ As, AB = AC

$\therefore \quad AD^2 = \frac{1}{4}AB^2$

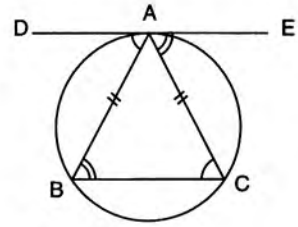
$\Rightarrow \quad AP \times AB = \frac{1}{4}AB^2$ $[\because AD^2 = AP \times AB]$

$\Rightarrow \quad AP = \quad AB$ and **AB = 4 × AP** **Hence the requred result.**

14 The given figure shows an isosceles triangle ABC inscribed in a circle such that AB = AC. If DAE is a tangent to the circle at point A, prove that DE is parallel to BC.



Solution :



Since, the angle between a tangent and a chord through the point of contant is equal to an angle in the alternate segment,

∴	$\angle DAB$	$= \angle ACB$	...I	[DAE in a tangent and AB is a chord]
	Since, AB	$= AC$		[Given]
∴	$\angle ACB$	$= \angle ABC$	...II	[Angles opp. to equal sides of a Δ are equal]
⇒	$\angle DAB$	$= \angle ABC$	...II	[From I and II]

But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

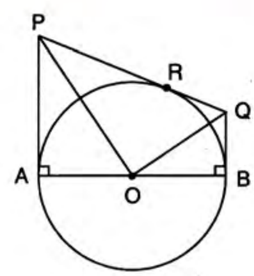
∴ **DAE is parallel to BC**

Hence Proved.

15 AB is the diameter of a circle with centre O. A line PQ touches the given circle at point R and cuts the tangents to the circle through A and B at points P and Q respectively. Prove that : ∠POQ = 90°.

Solution :

According to the given statement, the figure will be as shown alongside :



Join OR.

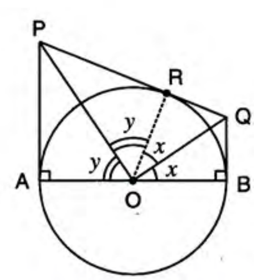
Now, show that Δ OBQ ≡ Δ ORQ

⇒ $\angle BOQ = \angle ROQ = x$ (let)

Then, show that Δ OAP ≡ Δ ORP

⇒ $\angle AOP = \angle ROP = y$ (let)

Since, AOB is a straight line, therefore $\angle AOB = 180°$



⇒ $x + x + y + y = 180°$

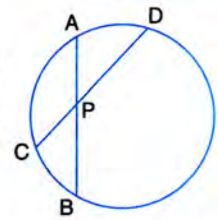
i.e. $2x + 2y = 180°$

⇒ $x + y = 90°$

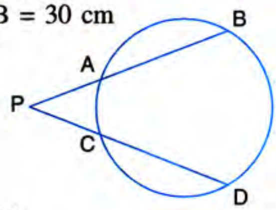
⇒ **$\angle POQ = 90°$**

Hence Proved.

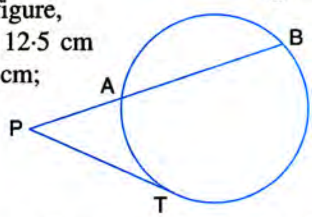
1. (i) In the given figure,
$3 \times CP = PD = 9$ cm
and $AP = 4{\cdot}5$ cm.
Find BP.



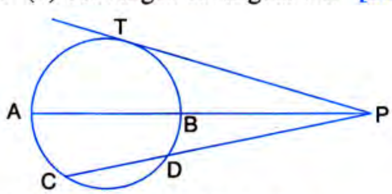
(ii) In the given figure,
$5 \times PA = 3 \times AB = 30$ cm
and $PC = 4$ cm.
Find CD.



(iii) In the given figure,
tangent $PT = 12{\cdot}5$ cm
and $PA = 10$ cm;
find AB.



2. In the given figure, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. $CD = 7{\cdot}8$ cm, $PD = 5$ cm, $PB = 4$ cm. Find :
(i) AB. (ii) the length of tangent PT. **[2014]**

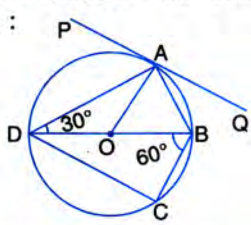


3. In the following figure, PQ is the tangent to the circle at A, DB is the diameter and O is the centre of the circle. If $\angle ADB = 30°$ and $\angle CBD = 60°$, calculate :

(i) $\angle QAB$,

(ii) $\angle PAD$,

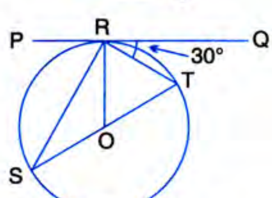
(iii) $\angle CDB$.



4. If PQ is a tangent to the circle at R; calculate :

(i) $\angle PRS$,

(ii) $\angle ROT$.



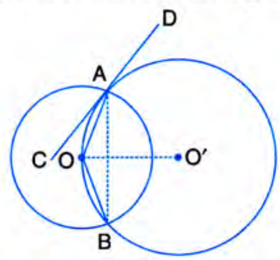
Given O is the centre of the circle and angle $TRQ = 30°$.

5. AB is the diameter and AC is a chord of a circle with centre O such that angle $BAC = 30°$. The tangent to the circle at C intersects AB produced in D. Show that : BC = BD.

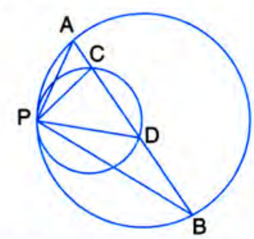
6. Tangent at P to the circumcircle of triangle PQR is drawn. If this tangent is parallel to side, QR show that Δ PQR is isosceles.

7. Two circles with centres O and O′ are drawn to intersect each other at points A and B.

Centre O of one circle lies on the circumference of the other circle and CD is drawn tangent to the circle with centre O′ at A. Prove that OA bisects angle BAC.



8. Two circles touch each other internally at a point P. A chord AB of the bigger circle intersects the other circle in C and D. Prove that : $\angle CPA = \angle DPB$.



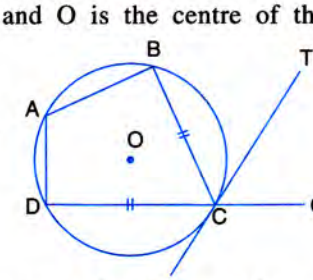
Draw tangent at point P.

9. In a cyclic quadrilateral ABCD, the diagonal AC bisects the angle BCD. Prove that the diagonal BD is parallel to the tangent to the circle at point A.

10. In the figure, ABCD is a cyclic quadrilateral with BC = CD. TC is tangent to the circle at point C and DC is produced to point G. If $\angle BCG = 108°$ and O is the centre of the circle, find :

(i) angle BCT

(ii) angle DOC

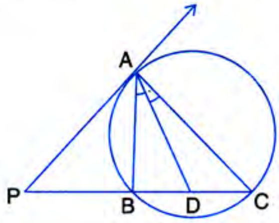


11. Two circles intersect each other at points A and B. A straight line PAQ cuts the circles at P and Q. If the tangents at P and Q intersect

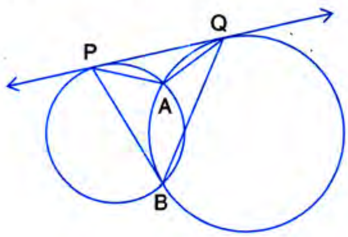
at point T; show that the points P, B, Q and T are concyclic.

12. In the figure; PA is a tangent to the circle, PBC is secant and AD bisects angle BAC. Show that triangle PAD is an isosceles triangle. Also, show that :

$\angle CAD = \frac{1}{2} [\angle PBA - \angle PAB]$

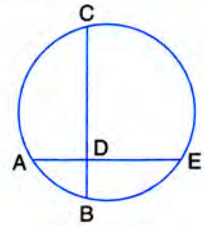


13. Two circles intersect each other at points A and B. Their common tangent touches the circles at points P and Q as shown in the figure. Show that the angles PAQ and PBQ are supplementary.



[2000]

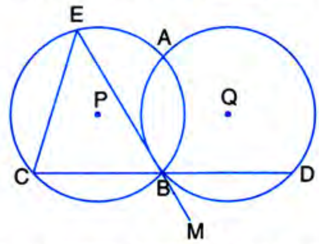
14. In the figure, chords AE and BC intersect each other at point D.



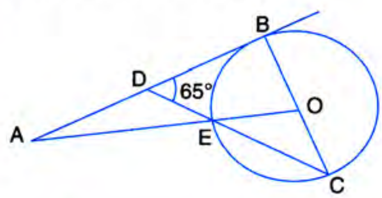
(i) If $\angle CDE = 90°$, AB = 5 cm, BD = 4 cm and CD = 9 cm; find DE.

(ii) If AD = BD, show that : AE = BC.

15. Circles with centres P and Q intersect at points A and B as shown in the figure. CBD is a line segment and EBM is tangent to the circle, with centre Q, at point B. If the circles are congruent; show that : CE = BD.



16. In the adjoining figure, O is the centre of the circle and AB is a tangent to it at point B. $\angle BDC = 65°$. Find $\angle BAO$.



[2010]

EXERCISE 18(C)

1. Prove that, of any two chords of a circle, the greater chord is nearer to the centre.

2. OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O.

   (i) If the radius of the circle is 10 cm, find the area of the rhombus.

   (ii) If the area of the rhombus is $32\sqrt{3}$ cm^2 find the radius of the circle.

3. Two circles with centres A and B, and radii 5 cm and 3 cm, touch each other internally. If the perpendicular bisector of the segment AB meets the bigger circle in P and Q; find the length of PQ.

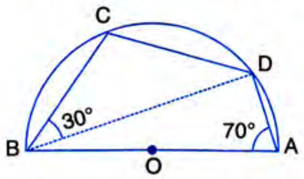
4. Two chords AB and AC of a circle are equal. Prove that the centre of the circle lies on the bisector of angle BAC.

   Show that the bisector of angle BAC is a perpendicular bisector of chord BC

5. The diameter and a chord of a circle have a common end-point. If the length of the diameter is 20 cm and the length of the chord is 12 cm, how far is the chord from the centre of the circle ?

6. ABCD is a cyclic quadrilateral in which BC is parallel to AD, angle ADC = 110° and angle BAC = 50°. Find angle DAC and angle DCA.

7. In the given figure, C and D are points on the semi-circle described on AB as diameter. Given angle BAD = 70° and angle DBC = 30°, calculate angle BDC.

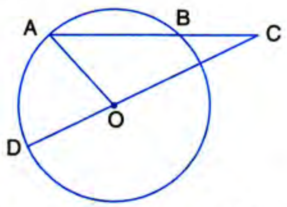


8. In cyclic quadrilateral ABCD, ∠A = 3 ∠C and ∠D = 5 ∠B. Find the measure of each angle of the quadrilateral.

9. Show that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.

10. Bisectors of vertex angles A, B and C of a triangle ABC intersect its circumcircle at the points D, E and F respectively. Prove that angle EDF = $90° - \frac{1}{2}\angle A$.

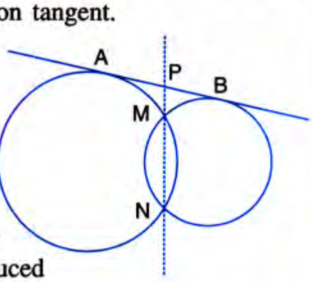
11. In the figure, AB is the chord of a circle with centre O and DOC is a line segment such that BC = DO. If ∠C = 20°, find angle AOD.



12. Prove that the perimeter of a right triangle is equal to the sum of the diameter of its incircle and twice the diameter of its circumcircle.

13. P is the mid-point of an arc APB of a circle. Prove that the tangent drawn at P will be parallel to the chord AB.

14. In the given figure, MN is the common chord of two intersecting circles and AB is their common tangent.



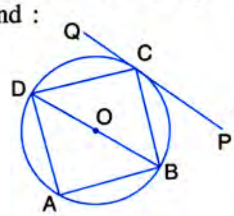
Prove that the line NM produced bisects AB at P.

15. In the given figure, ABCD is a cyclic-quadrilateral, PQ is tangent to the circle at point C and BD is its diameter. If ∠DCQ = 40° and ∠ABD = 60°, find :

(i) ∠DBC

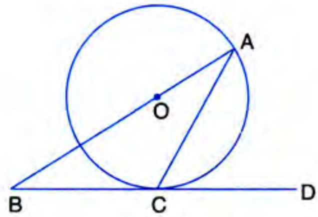
(ii) ∠BCP

(iii) ∠ADB



16. The given figure shows a circle with centre O and BCD is tangent to it at C. Show that :

∠ACD + ∠BAC = 90°.



17. ABC is a right triangle with angle B = 90°. A circle with BC as diameter meets hypotenuse AC at point D. Prove that :

(i) AC × AD = AB^2

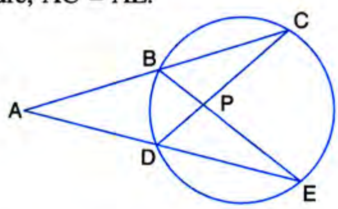
(ii) BD^2 = AD × DC.

18. In the given figure, AC = AE.

Show that :

(i) CP = EP

(ii) BP = DP.



19. ABCDE is a cyclic pentagon with centre of its circumcircle at point O such that AB = BC = CD and angle ABC = 120°.

Calculate :

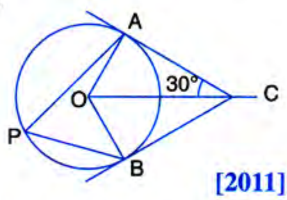
(i) ∠BEC (ii) ∠BED

20. In the given figure, O is the centre of the circle. Tangents at A and B meet at C. If ∠ACO = 30°, find :

(i) ∠BCO

(ii) ∠AOB

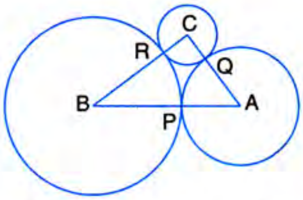
(iii) ∠APB



[2011]

21. ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each

other with the vertices as their centres. Find the radii of the three circles.



[2011]

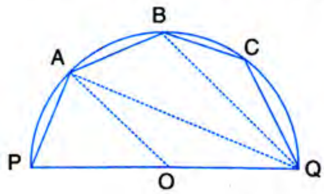
22. In a square ABCD, its diagonals AC and BD intersect each other at point O. The bisector of angle DAO meets BD at point M and the bisector of angle ABD meets AC at N and AM at L. Show that :

(i) $\angle ONL + \angle OML = 180°$

(ii) $\angle BAM = \angle BMA$

(iii) ALOB is a cyclic quadrilateral.

23. The given figure shows a semi-circle with centre O and diameter PQ. If PA = AB and $\angle BCQ = 140°$; find measures of angles PAB and AQB. Also, show that AO is parallel to BQ.



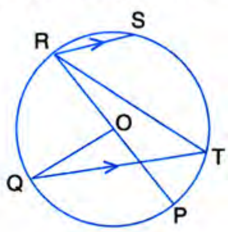
24. The given figure shows a circle with centre O such that chord RS is parallel to chord QT, angle PRT = 20° and angle POQ = 100°. Calculate :

(i) angle QTR

(ii) angle QRP

(iii) angle QRS

(iv) angle STR

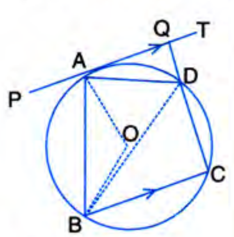


25. In the given figure, PAT is tangent to the circle with centre O, at point A on its circumference and is parallel to chord BC. If CDQ is a line segment, show that :

(i) $\angle BAP = \angle ADQ$

(ii) $\angle AOB = 2\angle ADQ$

(iii) $\angle ADQ = \angle ADB$.



26. AB is a line segment and M is its mid-point. Three semi-circles are drawn with AM, MB and AB as diameters on the same side of the line AB. A circle with radius r unit is drawn so that it touches all the three semi-circles. Show that : $AB = 6 \times r$

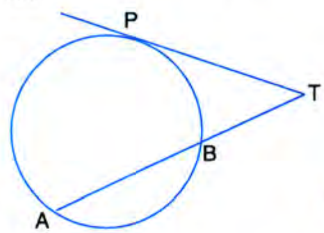
27. TA and TB are tangents to a circle with centre O from an external point T. OT intersects the circle at point P. Prove that AP bisects the angle TAB.

28. Two circles intersect in points P and Q. A secant passing through P intersects the circles in A and B respectively. Tangents to the circles at A and B intersect at T. Prove that A, Q, B and T lie on a circle.

29. Prove that any four vertices of a regular pentagon are concyclic (lie on the same circle).

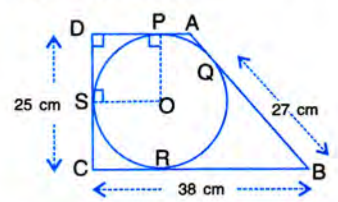
30. Chords AB and CD of a circle when extended meet at point X. Given AB = 4 cm, BX = 6 cm and XD = 5 cm, calculate the length of CD. **[2000]**

31. In the given figure, find TP if AT = 16 cm and AB = 12 cm.



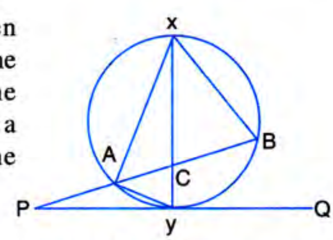
[2007]

32. In the following figure, a circle is inscribed in the quadrilateral ABCD.



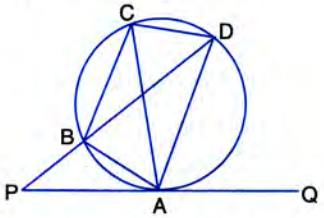
If BC = 38 cm, QB = 27 cm, DC = 25 cm and that AD is perpendicular to DC, find the radius of the circle.

33. In the given figure, XY is the diameter of the circle and PQ is a tangent to the circle at Y.



If $\angle AXB = 50°$ and $\angle ABX = 70°$, find $\angle BAY$ and $\angle APY$.

34. In the given figure, QAP is the tangent at point A and PBD is a straight line.



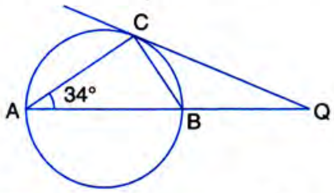
If ∠ACB = 36° and ∠APB = 42°, find :

(i) ∠BAP (ii) ∠ABD

(iii) ∠QAD (iv) ∠BCD

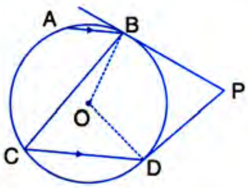
35. In the given figure, AB is the diameter. The tangent at C meets AB produced at Q.

If ∠CAB = 34°, find :



(i) ∠CBA (ii) ∠CQB **[2006]**

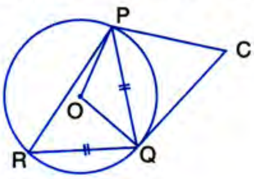
36. In the given figure, O is the centre of the circle. The tangents at B and D intersect each other at point P.



If AB is parallel to CD and ∠ABC = 55°, find :

(i) ∠BOD (ii) ∠BPD

37. In the following figure, PQ = QR, ∠RQP = 68°, PC and CQ are tangents to the circle with centre O.

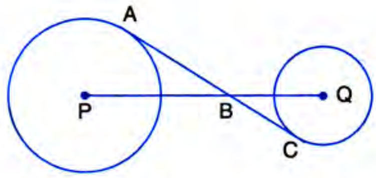


Calculate the values of :

(i) ∠QOP (ii) ∠QCP **[2008]**

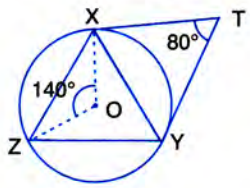
38. In two concentric circles, prove that all chords of the outer circle, which touch the inner circle, are of equal length.

39. In the figure, given below, AC is a transverse common tangent to two circles with centres P and Q and of radii 6 cm and 3 cm repectively.



Given that AB = 8 cm, calculate PQ.

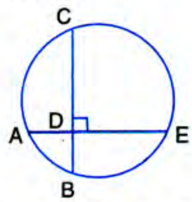
40. In the figure, given below, O is the centre of the circumcircle of triangle XYZ.



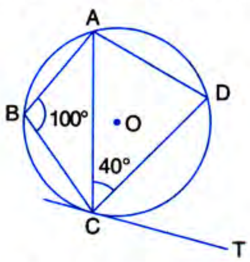
Tangents at X and Y intersect at point T. Given ∠XTY = 80° and ∠XOZ = 140°, calculate the value of ∠ZXY.

41. In the given figure, AE and BC intersect each other at point D.

If ∠CDE = 90°, AB = 5 cm, BD = 4 cm and CD = 9 cm, find AE.



42. In the given circle with centre O, ∠ABC = 100°, ∠ACD = 40° and CT is a tangent to the circle at C. Find ∠ADC and ∠DCT.



[2013]

43. In the figure given below, O is the centre of the circle and SP is a tangent. If ∠SRT = 65°, find the values of x, y and z. **[2015]**