# 19 Constructions

## (Circles)

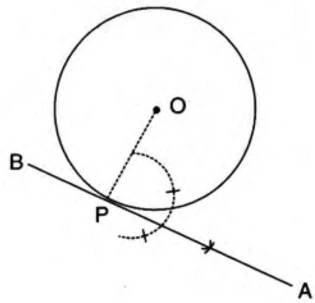
### 19.1 Construction of Tangents to a Given Circle :

**Construction 1 :** *To construct a tangent to a given circle through a point on its circumference.*

Let the centre of the given circle be O and P be any point on its circumference.

**Steps :**

1. Join O and P.
2. Draw line APB making angle of 90° with OP, *i.e.* $\angle OPA = 90°$.




**APB is the required tangent** to the given circle through a point P on its circumference.

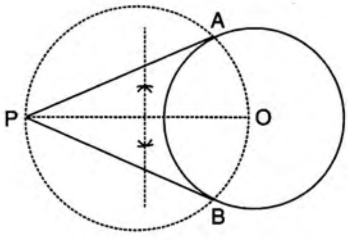
**Remember :** Angle between the radius and the tangent at the point of contact is 90°.

**Construction 2 :** *To construct tangents to a given circle from an exterior point.*

Let the centre of the given circle be O and P be an exterior point, *i.e.* P lies outside the circle.

**Steps :**

1. Join P and O.
2. Draw a circle with OP as diameter which cuts the given circle at points A and B.
3. Join PA and PB.




**PA and PB are the required tangents** to the given circle from an exterior point P.

1. Tangents drawn to a circle from an exterior point are always equal in length *i.e.* PA = PB.
2. Since the angle of semi-circle is 90°, therefore, $\angle PAO = 90°$ and

   $PA^2 + OA^2 = OP^2$ [Pythagoras Theorem]

   or, $PA = \sqrt{OP^2 - OA^2} = PB$.

## 19.2 Construction of Circumscribed and Inscribed Circles of a Triangle:

1. To construct a circle **circumscribing** a given triangle (say, ΔABC), draw the perpendicular bisectors of any two sides of the triangle. Let these perpendicular bisectors meet at point O. Taking point O as centre and OA or OB or OC as radius, draw a circle which will pass through all the three vertices of the triangle.

   Here, point O is called the **circumcentre** of the triangle and OA = OB = OC is called its **circumradius**.

2. To construct an **inscribed circle** in a given triangle (say, ΔABC); draw the bisectors of any two angles of the triangle. Let these angle bisectors meet at point I. From the point I, draw ID perpendicular to any side of the given ΔABC. Now with I as centre and ID as radius, draw a circle which will touch all the three sides of the given ΔABC.

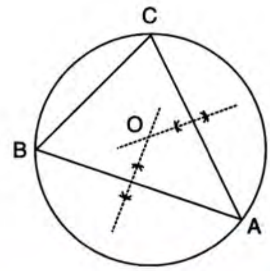
   Here, point I is called **incentre** of the triangle and ID is called **in radius**.

3. Whether the triangle is regular shaped (equilateral triangle) or not, the methods for above constructions will be the same.

**Construction 3 :** *To construct a circumscribing circle of a triangle.*

Let ABC be the given triangle.

**Steps :**




1. Draw the perpendicular bisectors of any two sides of the triangle.

   Let the perpendicular bisectors of AB and AC be drawn which meet at point O.

2. Taking O as the centre and radius equal to OA (or, OB or, OC) draw a circle.

   **The circle so obtained is the required circle.**

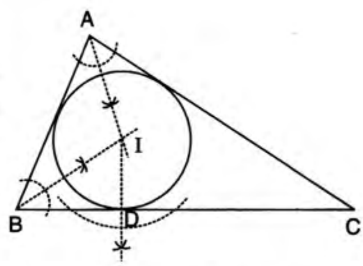
The perpendicular bisectors of the sides of a triangle are concurrent, *i.e.* they meet at one point (point O in the above construction).

This point O, where the perpendicular bisectors of the sides of a triangle meet, is equidistant from the vertices of the triangle *i.e.* OA = OB = OC and is called the **circumcentre** of the triangle.

**Construction 4 :** *To construct an inscribed circle of a triangle.*

Let ABC be the given triangle.

**Steps :**




1. Draw the bisectors of any two angles of the triangle. Let the bisectors of angles A and B be drawn and they meet at I.

2. From I, drop perpendicular on any side of the triangle. Let ID be the perpendicular drawn from I to side BC.

3. With I as centre and ID as radius, draw a circle which will touch all the three sides of the triangle.

   The circle so obtained is the required circle.

The angle bisectors of a triangle are concurrent, *i.e.* they pass through the same point (the point I in the above construction).

The point I, where the angle bisectors of the triangle meet, is equidistant from the sides of the triangle and is called **incentre** of the triangle.

## 19.3 Circumscribing and Inscribing a Circle on Regular Hexagon :

1. To construct a circle **circumscribing** a given regular hexagon; draw the perpendicular bisectors of any two sides of it. Taking the point of intersection of these perpendicular bisectors as centre and its distance from any vertex of the given regular hexagon as radius; draw a circle which will pass through all the vertices of the given regular hexagon.
2. To construct an **inscribing** circle in a given regular hexagon; draw the bisectors of any two angles of it. From the point of intersection of these angle bisectors, draw perpendicular to any side of the given regular hexagon.

   With point of intersection of the angle bisectors as centre and radius equal to the length of perpendicular, draw a circle. This circle will touch each side of the given regular hexagon.
3. If the given hexagon is not regular, it is not always possible to draw is circumscribing or inscribing circle.

**Construction 5 :** *To construct a circumscribing circle of a given regular hexagon.*

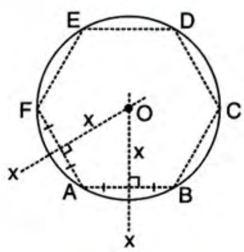
Let each side of the given regular hexagon be 4 cm.

Each interior angle of the regular hexagon

$$= \left(\frac{2n - 4}{n}\right) \times 90° = \left(\frac{2 \times 6 - 4}{6}\right) \times 90° = 120°$$

**Steps :**

1. Using the given data, construct the regular hexagon ABCDEF with each side equal to 4 cm.
2. Draw the perpendicular bisectors of sides AB and AF which intersect each other at point O.
3. With O as centre and OA as radius draw a circle which will pass through all the vertices of the regular hexagon ABCDEF.




***The circle so obtained is the required circle circumscribing the given regular hexagon.***

**Alternative Method :**

Whenever a circle circumscribes a given regular hexagon, its radius is always equal to the length of the side of the regular hexagon.

**Steps :**

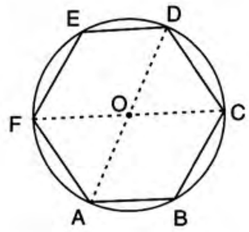
1. Using the given data, construct a regular hexagon ABCDEF with each side equal to 4 cm.

2. Draw any two main diagonals of the given regular polygon.

   Here, main diagonals AD and FC are drawn which meet at point O.

3. Taking O as centre and OA as radius draw a circle which will pass through all the vertices of the regular hexagon ABCDEF.

   *The circle so obtained is the required circle circumscribing the given regular hexagon.*



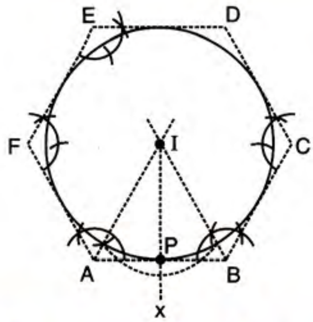

**Construction 6 :** *To construct an inscribing circle of a given regular hexagon.*

Let each side of the given regular hexagon be 4.6 cm.

> Each interior angle of the regular hexagon
>
> $$= \left(\frac{2 \times 6 - 4}{6}\right) \times 90° = 120°$$

**Steps :**

1. Using the given data, construct the regular hexagon ABCDEF with each side equal to 4·6 cm.
2. Draw the bisectors of interior angles at A and at B which intersect each other at point I.
3. From point I, draw IP perpendicular to AB.
4. With I as centre and IP as radius, draw a circle which will touch all the sides of the regular hexagon drawn.




***The circle so obtained is the required inscribing circle of the given regular hexagon.***

## EXERCISE 19

1. Draw a circle of radius 3 cm. Mark a point P at a distance of 5 cm from the centre of the circle drawn. Draw two tangents PA and PB to the given circle and measure the length of each tangent.
2. Draw a circle of diameter 9 cm. Mark a point at a distance of 7·5 cm from the centre of the circle. Draw tangents to the given circle from this exterior point. Measure the length of each tangent.
3. Draw a circle of radius 5 cm. Draw two tangents to this circle so that the angle between the tangents is 45°.

> Draw two radii of the circle so that they make an angle equal to 180° – 45° = 135° at the centre of the circle.

4. Draw a circle of radius 4·5 cm. Draw two tangents to this circle so that the angle between the tangents is 60°.
5. Using ruler and compasses only, draw an equilateral triangle of side 4·5 cm and draw its circumscribed circle. Measure the radius of the circle.
6. Using ruler and compasses only,
   (i) Construct triangle ABC, having given BC = 7 cm, AB – AC = 1 cm and ∠ABC = 45°.
   (ii) Inscribe a circle in the Δ ABC constructed in (i) above.
7. Using ruler and compasses only, draw an equilateral triangle of side 5 cm. Draw its inscribed circle. Measure the radius of the circle.

8. Using ruler and compasses only,
   (i) Construct a triangle ABC with the following data :
   Base AB = 6 cm, BC = 6·2 cm and $\angle$CAB = 60°.
   (ii) In the same diagram, draw a circle which passes through the points A, B and C and mark its centre O.
   (iii) Draw a perpendicular from O to AB which meets AB in D.
   (iv) Prove that : AD = BD.
9. Using ruler and compasses only construct a triangle ABC in which BC = 4 cm, $\angle$ACB = 45° and perpendicular from A on BC is 2·5 cm. Draw a circle circumscribing the triangle ABC.
10. Perpendicular bisectors of the sides AB and AC of a triangle ABC meet at O.
    (i) What do you call the point O ?
    (ii) What is the relation between the distances OA, OB and OC ?
    (iii) Does the perpendicular bisector of BC pass through O ?
11. The bisectors of angles A and B of a scalene triangle ABC meet at O.
    (i) What is the point O called ?
    (ii) OR and OQ are drawn perpendiculars to AB and CA respectively. What is the relation between OR and OQ ?
    (iii) What is the relation between angle ACO and angle BCO ?
12. (i) Using ruler and compasses only, construct a triangle ABC in which AB = 8 cm, BC = 6 cm and CA = 5 cm.
    (ii) Find its incentre and mark it I.
    (iii) With I as centre, draw a circle which will cut off 2 cm chords from each side of the triangle.
13. Construct an equilateral triangle ABC with side 6 cm. Draw a circle circumscribing the triangle ABC.
14. Construct a circle, inscribing an equilateral triangle with side 5·6 cm.
15. Draw a circle circumscribing a regular hexagon with side 5 cm.
16. Draw an inscribing circle of a regular hexagon of side 5·8 cm.
17. Construct a regular hexagon of side 4 cm. Construct a circle circumscribing the hexagon. **[2010]**
18. Draw a circle of radius 3·5 cm. Mark a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent. **[2011]**
19. Construct a triangle ABC in which base BC = 5·5 cm, AB = 6 cm and $\angle$ABC = 120°.
    (i) Construct a circle circumscribing the triangle ABC.
    (ii) Draw a cyclic quadrilateral ABCD so that D is equidistant from B and C. **[2012]**
20. Using a ruler and compasses only :
    (i) Construct a triangle ABC with the following data :
    AB = 3·5 cm, BC = 6 cm and $\angle$ABC = 120°.
    (ii) In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC.
    (iii) Measure $\angle$BCP. **[2013]**
21. Construct a $\Delta$ABC with BC = 6·5 cm, AB = 5·5 cm, AC = 5 cm. Construct the incircle of the triangle. Measure and record the radius of the incircle. **[2014]**
22. Construct a triangle ABC with AB = 5·5 cm, AC = 6 cm and $\angle$BAC = 105°. Hence :
    (i) Construct the locus of points equidistant from BA and BC.
    (ii) Construct the locus of points equidistant from B and C.
    (iii) Mark the point which satisfies the above two loci as P. Measure and write the length of PC. **[2015]**