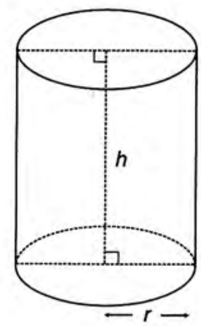
20 Cylinder, Cone and Sphere

(Surface Area and Volume)

UNIT 4 : Mensuration

20.1 Cylinder :

A **solid** which has *uniform circular cross-section*, is called a ***cylinder*** **(or a circular cylinder)**.



Let r be the radius of circular cross-section and h the height of the cylinder; then

1 Area of cross-section $= \pi r^2$.

2. Perimeter (circumference) of cross-section $= 2\pi r$

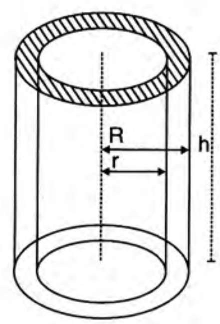
3. Curved surface area = Perimeter of cross-section × height $= 2\pi rh$.

4. Total surface area = Curved surface area + 2 (Area of cross-section) $= 2\pi rh + 2(\pi r^2) = 2\pi r(h + r)$.

5. Volume = Area of cross-section × height (or, length) $= \pi r^2 h$.

20.2 Hollow Cylinder :

Let R be the external radius of a hollow cylinder, r its internal radius and h its height or length; then



1. Thickness of its wall $= R - r$

2. Area of cross-section $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

3. External curved surface $= 2\pi Rh$

4. Internal curved surface $= 2\pi rh$

5. Total surface area = External curved surface area + Internal curved surface area + 2 (Area of cross-section) $= 2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

6. Volume of material = External volume – Internal volume $= \pi R^2 h - \pi r^2 h$ $= \pi(R^2 - r^2)\,h$.

1 The area of the curved surface of a cylinder is 4,400 cm^2 and the circumference of its base is 110 cm. Find :

(i) the height of the cylinder, (ii) the volume of the cylinder.

Solution :

(i) Given, $2\pi rh = 4400 \text{ cm}^2$ and $2\pi r = 110$ cm

$$\therefore \frac{2\pi rh}{2\pi r} = \frac{4400}{110} \text{cm} \Rightarrow \mathbf{h = 40 \text{ cm}}$$ **Ans.**

(ii) Since, $2\pi r = 110$

$$\therefore r = \frac{110}{2\pi} = \frac{110 \times 7}{2 \times 22} \text{cm} = \frac{35}{2} \text{ cm}$$

and, **volume** $= \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 40 \text{ cm}^3$ [Since, volume = $\pi r^2 h$]

$= \mathbf{38{,}500 \text{ cm}^3}$ **Ans.**

2 The barrel of a fountain-pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used up when writing 310 words on an average. How many words would use up a bottle of ink containing one-fifth of a litre ?

Answer correct to the nearest 100 words

Solution :

One-fifth of a litre $= \frac{1}{5} \times 1000 \text{ cm}^3 = 200 \text{ cm}^3$ [$\because$ 1 litre = 1000 cm^3]

Volume of ink in the barrel $= \pi r^2 h$

$= \frac{22}{7} \times (0{\cdot}25)^2 \times 7 \text{ cm}^3$ [Since, $r = \frac{5}{2}$ mm = 0·25cm]

$= 1{\cdot}3750 \text{ cm}^3$

$\therefore$ 1·3750 cm^3 of ink is used in writing 310 words

$\Rightarrow$ 1 cm^3 of ink is used in writing $\frac{310}{1{\cdot}3750}$ words

and 200 cm^3 of ink is used in writing $\frac{310}{1{\cdot}3750} \times 200$ words

= 45091 words

= **45100 words** [To the nearest 100 words] **Ans.**

3 A cylindrical tube, open at both ends, is made of metal. The internal diameter of the tube is 11·2 cm and its length is 21 cm. The metal everywhere is 0·4 cm in thickness. Calculate the volume of the metal correct to one place of decimal.

Solution :

Since, internal radius (r) $= \frac{11 \cdot 2}{2}$ cm = 5·6 cm

$\Rightarrow$ External radius (R) = 5·6 cm + 0·4 cm = 6·0 cm

$\therefore$ **Volume of metal** $= \pi(R^2 - r^2)\, h$

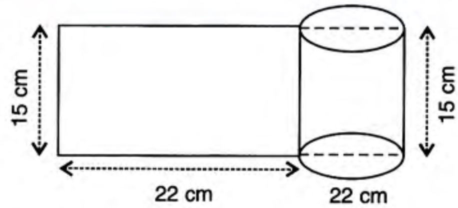
$= \frac{22}{7} [(6)^2 - (5 \cdot 6)^2] \times 21 \text{ cm}^3$ [Given, h = 21 cm]

$= 306 \cdot 24 \text{ cm}^3 =$ **306·2 cm³** **Ans.**

4 Find the volume of the largest cylinder formed when a rectangular piece of paper 22 cm by 15 cm is rolled along its longer side.

Solution :

It is clear from the figure, drawn alongside, that the circumference of the cross-section of the cylinder formed is 22 cm and its height is 15 cm.



Let the radius of the cylinder = r cm

$\therefore 2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{7}{2}$ cm.

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15 \text{ cm}^3 =$ **577·5 cm³** **Ans.**

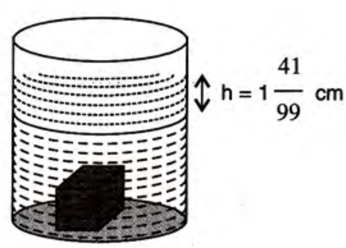
5 When a metal cube is completely submerged in water contained in a cylindrical vessel with diameter 30 cm, the level of water rises by $1\frac{41}{99}$ cm.

Find : (i) the length of edge of the cube. (ii) the total surface area of the cube.

Solution :

(i) $\therefore$ Radius (r) of the vessel $= \frac{30}{2}$ cm

= 15 cm



and, rise in level (h) of the water $= 1\frac{41}{99}$ cm

$= \frac{140}{99}$ cm

$\therefore$ Volume of water that rises $= \pi r^2 h$

$= \frac{22}{7} \times (15)^2 \times \frac{140}{99} \text{ cm}^3 = 1000 \text{ cm}^3$

Let the length of edge of the cube = a cm

$\therefore$ Its volume = a^3 cm³

Clearly, volume of the cube submerged = vol. of water that rises by $1\frac{41}{99}$ cm

$\Rightarrow \quad a^3 = 1000 = 10^3$

$\therefore \quad a = 10$

$\Rightarrow$ **The length of the edge of the cube = 10 cm** **Ans.**

(ii) **Total surface area of the cube** = $6a^2 = 6 \times 10^2$ cm^2 = **600 cm^2** **Ans.**

EXERCISE 20(A)

1. The height of a circular cylinder is 20 cm and the radius of its base is 7 cm. Find :
   (i) the volume
   (ii) the total surface area.

2. The inner radius of a pipe is 2·1 cm. How much water can 12 m of this pipe hold ?

3. A cylinder of circumference 8 cm and length 21 cm rolls without sliding for $4\frac{1}{2}$ seconds at the rate of 9 complete rounds per second.

   Find :
   (i) the distance travelled by the cylinder in $4\frac{1}{2}$ seconds, and
   (ii) the area covered by the cylinder in $4\frac{1}{2}$ seconds.

> (i) Distance travelled by the cylinder in 1 round = its circumference.
> (ii) Area covered by the cylinder in 1 round = its curved surface area.

4. How many cubic metres of earth must be dug out to make a well 28 m deep and 2·8 m in diameter ? Also, find the cost of plastering its inner surface at ₹ 4.50 per sq. metre.

5. What length of solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of external diameter 20 cm, 0·25 cm thick and 15 cm long ?

6. A cylinder has a diameter of 20 cm. The area of the curved surface is 100 cm^2 (sq. cm). Find :
   (i) the height of the cylinder correct to one decimal place.
   (ii) the volume of the cylinder correct to one decimal place.

7. A metal pipe has a bore (inner diameter) of 5 cm. The pipe is 5 mm thick all round. Find the weight, in kilogram, of 2 metres of the pipe if 1 cm^3 of the metal weighs 7·7 g.

8. A cylindrical container with diameter of base 42 cm contains sufficient water to submerge a rectangular solid of iron with dimensions 22 cm × 14 cm × 10·5 cm. Find the rise in the level of the water when the solid is submerged.

9. A cylindrical container with internal radius of its base 10 cm, contains water up to a height of 7 cm. Find the area of the wet surface of the cylinder.

10. Find the total surface area of an open pipe of length 50 cm, external diameter 20 cm and internal diameter 6 cm.

11. The height and the radius of the base of a cylinder are in the ratio 3 : 1. If its volume is1029 π cm^3; find its total surface area.

12. The radius of a solid right circular cylinder increases by 20% and its height decreases by 20%. Find the percentage change in its volume.

> Let the radius = 100 cm
> and the height = 100 cm
> $\therefore$ Volume = $\pi r^2 h = \pi(100)^2 \times 100$ cm^3
> $= 1000000\ \pi$ cm^3
> New radius = 120 cm
> and the new height = 80 cm
> $\Rightarrow$ New volume = $\pi(120)^2 \times 80$ cm^3
> $= 11,52,000\ \pi$ cm^3
> Increase in volume = $1,52,000\ \pi$ cm^3
> Percentage change (increase) in volume
> $= \frac{\text{Increase in Vol.}}{\text{Original Vol.}} \times 100\%$
> $= \frac{152000\pi}{1000000\pi} \times 100\% = 15{\cdot}2\%$

13. The radius of a solid right circular cylinder decreases by 20% and its height increases by 10%. Find the percentage change in its :

(i) volume (ii) curved surface area

14. Find the minimum length in cm and correct to nearest whole number of the thin metal sheet required to make a hollow and closed cylindrical box of diameter 20 cm and height 35 cm. Given that the width of the metal sheet is 1 m. Also, find the cost of the sheet at the rate of ₹ 56 per m.

Find the area of metal sheet required, if 10% of it is wasted in cutting, overlapping, etc.

15. 3080 cm^3 of water is required to fill a cylindrical vessel completely and 2310 cm^3 of water is required to fill it upto 5 cm below the top. Find :

(i) radius of the vessel.

(ii) height of the vessel.

(iii) wetted surface area of the vessel when it is half-filled with water.

16. Find the volume of the largest cylinder formed when a rectangular piece of paper 44 cm by 33 cm is rolled along it :

(i) shorter side. (ii) longer side.

17. A metal cube of side 11 cm is completely submerged in water contained in a cylindrical vessel with diameter 28 cm. Find the rise in the level of water.

18. A circular tank of diameter 2 m is dug and the earth removed is spread uniformly all around the tank to form an embankment 2 m in width and 1·6 m in height. Find the depth of the circular tank.

19. The sum of the inner and the outer curved surfaces of a hollow metallic cylinder is 1056 cm^2 and the volume of material in it is 1056 cm^3. Find its internal and external radii. Given that the height of the cylinder is 21 cm.

> $2\pi Rh + 2\pi rh = 1056 \Rightarrow R + r = 8$ I
> $\pi R^2h - \pi r^2h = 1056 \Rightarrow R^2 - r^2 = 16$
> *i.e.* $(R + r)(R - r) = 16 \Rightarrow R - r = 2$ ... II
> Now solve equations I and II to get the values of external radius R and internal radius r.

20. The difference between the outer curved surface area and the inner curved surface area of a hollow cylinder is 352 cm^2. If its height is 28 cm and the volume of material in it is 704 cm^3; find its external curved surface area.

21. The sum of the height and the radius of a solid cylinder is 35 cm and its total surface area is 3080 cm^2; find the volume of the cylinder.

22. The total surface area of a solid cylinder is 616 cm^2. If the ratio between its curved surface area and total surface area is 1 : 2; find the volume of the cylinder.

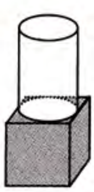
23. A cylindrical vessel of height 24 cm and diamater 40 cm is full of water. Find the exact number of small cylindrical bottles, each of height 10 cm and diameter 8 cm, which can be filled with this water.

24. Two solid cylinders, one with diameter 60 cm and height 30 cm and the other with radius 30 cm and height 60 cm, are melted and recasted into a third solid cylinder of height 10 cm. Find the diameter of the cylinder formed.

25. The total surface area of a hollow cylinder, which is open from both the sides, is 3575 cm^2; area of its base ring is 357·5 cm^2 and its height is 14 cm. Find the thickness of the cylinder.

> Use $\pi(R^2 - r^2) = 357·5$ to get $R^2 - r^2$.
> And, $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2) = 3575$ to get $R + r$.
> $\therefore$ Thickness $= R - r = \dfrac{R^2 - r^2}{R + r}$.

26. The given figure shows a solid formed of a solid cube of side 40 cm and a solid cylinder of radius 20 cm and height 50 cm attached to the cube as shown.



Find the volume and the total surface area of the whole solid [Take π = 3·14]

27. Two right circular solid cylinders have radii in the ratio 3 : 5 and heights in the ratio 2 : 3. Find the ratio between their :

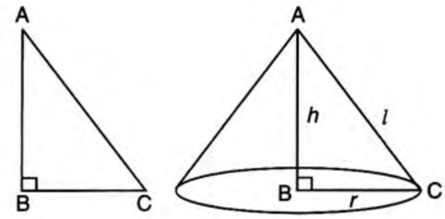
(i) curved surface areas.

(ii) volumes.

28. A closed cylindrical tank, made of thin iron-sheet, has diameter = 8·4 m and height 5·4 m. How much metal sheet, to the nearest m^2, is used in making this tank, if $\frac{1}{15}$ of the sheet actually used was wasted in making the tank ?

20.3 Cone :

The solid obtained on revolving a right-angled triangle about one of its sides (other than hypotenuse) is called ***a cone*** *or* ***a right circular cone.***



Let the right-angled triangle ABC be revolved about its side AB to form a cone; then AB is the height (h) of the cone formed, BC is the radius (r) of its base and AC is its slant height (l).

Clearly, $l^2 = h^2 + r^2$. [Using Pythagoras Theorem]

Also : 1. Volume $= \frac{1}{3}\pi r^2 h$

2. Curved or lateral surface area $= \pi rl$

3. Total surface area $=$ curved surface area + base area

$= \pi rl + \pi r^2 = \pi r(l + r)$

6 The radius of the base and the height of a right circular cone are 7 cm and 24 cm respectively. Find the volume and the total surface area of the cone.

Solution :

Given, radius (r) = 7 cm and height (h) = 24 cm

$\therefore \quad l^2 = h^2 + r^2 \Rightarrow l^2 = (24 \text{ cm})^2 + (7 \text{ cm})^2 \Rightarrow l = \sqrt{625} \text{ cm} = 25 \text{ cm}.$

$\therefore$ **The volume of the cone** $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24 \text{ cm}^3 =$ **1232 cm^3** **Ans.**

And, **the total surface area of the cone**

= Its curved surface area + area of base

$= \pi rl + \pi r^2$

$= \pi l(l + r)$

$= \frac{22}{7} \times 7 \times (25 + 7) \text{ cm}^2 =$ **704 cm^2** **Ans.**

7 A right circular cone is 3·6 cm high and radius of its base is 1·6 cm. It is melted and recasted into a right circular cone with radius of its base as 1·2 cm. Find its height.

Solution :

For 1st cone : $h = 3{\cdot}6$ cm and radius of base, $r = 1{\cdot}6$ cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times (1{\cdot}6)^2 \times 3{\cdot}6 \text{ cm}^3$

For 2nd cone : h = ? and $r = 1{\cdot}2$ cm

$\therefore$ Its volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times (1{\cdot}2)^2 \times h \text{ cm}^3$

According to the given statement :

Volume of 1st cone = Volume of 2nd cone

$$\Rightarrow \frac{1}{3} \times \pi \times (1{\cdot}6)^2 \times 3{\cdot}6 = \frac{1}{3} \times \pi \times (1{\cdot}2)^2 \times h$$

$$\Rightarrow h = \frac{1{\cdot}6 \times 1{\cdot}6 \times 3{\cdot}6}{1{\cdot}2 \times 1{\cdot}2} = \mathbf{6{\cdot}4 \text{ cm}}$$ **Ans.**

8 The ratio of the base area and the curved surface area of a conical tent is 40 : 41. If its height is 18 m, find the air capacity of the tent in terms of π.

Solution :

Let for the given tent, radius of base = r m, height = h m and slant height = l m

Given : $\frac{\text{Base area}}{\text{Curved surface area}} = \frac{40}{41} \Rightarrow \frac{\pi r^2}{\pi r l} = \frac{40}{41} \Rightarrow \frac{r}{l} = \frac{40}{41}$

Let $r = 40x$ m so $l = 41x$ m

$l^2 = h^2 + r^2 \Rightarrow (41x)^2 = h^2 + (40x)^2 \Rightarrow h = 9x$ m

Given : $h = 18$ m $\Rightarrow 9x = 18 \Rightarrow x = 2$

$\therefore$ Radius of base (r) = $40x$ m

$= 40 \times 2 \text{ m} = 80 \text{ m}$

$\therefore$ **Capacity of the tent** $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times (80)^2 \times 18$ cu m

$= \mathbf{38{,}400\pi}$ **cu m** **Ans.**

9 Find what length of canvas, 2 m in width, is required to make a conical tent 12 m in diameter and 63 m in slant height ?
Also, find the cost of the canvas at the rate of ₹ 150 per metre.

Solution :

Since the radius (r) of the conical tent $= \frac{12}{2}$ m = 6 m and its slant height (l) = 63 m.

$\therefore$ Area of canvas required = curved surface area of the tent

$= \pi r l$

$= \frac{22}{7} \times 6 \times 63 \text{ m}^2 = 1188 \text{ m}^2$

Let the length of the canvas required = x m

Since, length × width = area

$\Rightarrow$ $x \text{ m} \times 2 \text{ m} = 1188 \text{ m}^2$ *i.e.* $x = 594$ m

The length of canvas required = 594 m **Ans.**

And, **the cost of the canvas** = its length × rate

= 594 × ₹ 150 = **₹ 89,100** **Ans.**

10 A vessel, in the form of an inverted cone, is filled with water to the brim. Its height is 20 cm and diameter is 16·8 cm. Two equal solid cones are dropped in it so that they are fully submerged. As a result, one-third of the water in the original cone overflows. What is the volume of each of the solid cones submerged ? [2006]

Solution :

For the given conical vessel :

Height, h = 20 cm and radius, $r = \frac{16 \cdot 8}{2}$ cm = 8·4 cm

$\therefore$ Volume of water in the vessel = volume of the vessel

$= \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 8 \cdot 4 \times 8 \cdot 4 \times 20 \text{ cm}^3$

$= 1478 \cdot 4 \text{ cm}^3$

Given, on submerging two equal solid cones into it, one-third of the water overflows.

$\therefore$ Volume of two equal solid cones submerged

= volume of water that overflows

$= \frac{1}{3} \times 1478 \cdot 4 \text{ cm}^3 = 492 \cdot 8 \text{ cm}^3$

$\Rightarrow$ **Volume of each solid cone submerged**

$= \frac{492 \cdot 8}{2} \text{ cm}^3 =$ **$246 \cdot 4 \text{ cm}^3$** **Ans.**

11 Find the area of the canvas required to make a conical tent 14 m high and 96 m in diameter. Given that :

(i) 20% of the canvas is used in folds and stitchings.

(ii) canvas used in folds and stitchings is 20% of the curved surface area of the tent.

Solution :

For the conical tent :

Height, h = 14 m and radius, $r = \frac{96}{2}$ m = 48 m

$\therefore \quad l^2 = h^2 + r^2 \Rightarrow l^2 = 14^2 + 48^2$

$\Rightarrow l = 50$ m *i.e.* slant height = 50 m

Curved surface area of the tent $= \pi rl$

$= \frac{22}{7} \times 48 \times 50 \text{ m}^2 = \frac{52,800}{7} \text{ m}^2$

(i) Let the area of the canvas required = x m^2

$\therefore \quad x - 20\% \text{ of } x = \frac{52,800}{7} \text{ m}^2$

$\Rightarrow \quad x = \frac{66,000}{7} \text{ m}^2 = 9,428\frac{4}{7} \text{ m}^2$

i.e. **The area of canvas required = $9,428\frac{4}{7}$ m^2** **Ans.**

(ii) **Area of canvas required** = C.S.A. of the tent + 20% of it

$= \frac{52,800}{7} \text{ m}^2 + 20\% \text{ of } \frac{52,800}{7} \text{ m}^2$

$= \frac{63,360}{7} \text{ m}^2 = 9,051\frac{3}{7} \text{ m}^2$ **Ans.**

12 The capacity and the base area of a right circular conical vessel are 9856 cm^3 and 616 cm^2 respectively. Find the curved surface area of the vessel.

Solution :

Let radius of the base and height of the conical vessel be r cm and h cm respectively.

$\therefore \quad \pi r^2 = 616$ and $\frac{1}{3}\pi r^2 h = 9856$

$\Rightarrow \quad r^2 = \frac{616 \times 7}{22} = 196$ and $\frac{1}{3} \times 616 \times h = 9856$

$\Rightarrow \quad r = 14$ cm and $h = 48$ cm

If the slant height of the vessel be l cm, then $l^2 = h^2 + r^2$

$\Rightarrow \quad l = \sqrt{(48)^2 + (14)^2} = \sqrt{2500}$ cm = 50 cm

$\therefore$ **Curved surface area of the vessel**

$= \pi rl = \frac{22}{7} \times 14 \times 50 \text{ cm}^2 =$ **2200 cm^2** **Ans.**

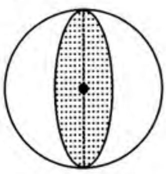
EXERCISE 20(B)

1. Find the volume of a cone whose slant height is 17 cm and radius of base is 8 cm.
2. The curved surface area of a cone is 12320 cm^2. If the radius of its base is 56 cm, find its height.
3. The circumference of the base of a 12 m high conical tent is 66 m. Find the volume of the air contained in it.
4. The radius and the height of a right circular cone are in the ratio 5 : 12 and its volume is 2512 cubic cm. Find the radius and slant height of the cone. (Take $\pi = 3{\cdot}14$)
5. Two right circular cones x and y are made. x having three times the radius of y and y having half the volume of x. Calculate the ratio between the heights of x and y.
6. The diameters of two cones are equal. If their slant heights are in the ratio 5 : 4, find the ratio of their curved surface areas.
7. There are two cones. The curved surface area of one is twice that of the other. The slant height of the latter is twice that of the former. Find the ratio of their radii.
8. A heap of wheat is in the form of a cone of diameter 16·8 m and height 3·5 m. Find its volume. How much cloth is required to just cover the heap ?
9. Find what length of canvas, 1·5 m in width, is required to make a conical tent 48 m in diameter and 7 m in height ? Given that 10% of the canvas is used in folds and stitchings. Also, find the cost of the canvas at the rate of ₹ 24 per metre.
10. A solid cone of height 8 cm and base radius 6 cm is melted and recast into identical cones, each of height 2 cm and diameter 1 cm. Find the number of cones formed.
11. The total surface area of a right circular cone of slant height 13 cm is 90 π cm^2. Calculate :
    (i) its radius in cm.
    (ii) its volume in cm^3. [Take $\pi = 3{\cdot}14$].
12. The area of the base of a conical solid is 38·5 cm^2 and its volume is 154 cm^3. Find curved surface area of the solid.
13. A vessel, in the form of an inverted cone, is filled with water to the brim. Its height is 32 cm and diameter of the base is 25·2 cm. Six equal solid cones are dropped in it, so that they are fully submerged. As a result, one-fourth of water in the original cone overflows. What is the volume of each of the solid cones submerged ?
14. The volume of a conical tent is 1232 m^3 and the area of the base floor is 154 m^2. Calculate the :
    (i) radius of the floor,
    (ii) height of the tent,
    (iii) length of the canvas required to cover this conical tent if its width is 2 m. **[2008]**

20.4 Sphere :

A sphere is a solid obtained on revolving a circle about any diameter of it.

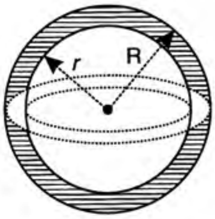
If radius of the circle revolved is r, then radius of the sphere is also r.



1. Volume of a sphere $= \frac{4}{3}\pi r^3$
   = Volume of material in the sphere.
2. Surface area of a sphere $= 4\pi r^2$

20.5 Spherical Shell :

It is the solid enclosed between two concentric spheres.



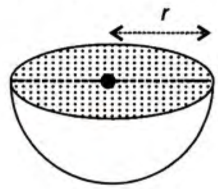
Let R be the external radius and r be the internal radius of a spherical shell,

then its volume $= \frac{4}{3}\pi(R^3 - r^3)$.

= Volume of material in the spherical shell.

20.6 Hemisphere :

*When a solid sphere is cut through its centre into two equal (identical) pieces; each piece is called **a hemisphere**.*



1. Volume of the hemisphere $= \frac{1}{2}$(volume of sphere)

$$= \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$$

2. Total surface area $= \frac{1}{2}$ (surface area of sphere) $+ \pi r^2$

$$= \frac{1}{2} \times 4\pi r^2 + \pi r^2 = 3\pi r^2$$

13 If the surface area of a sphere is 616 cm^2, find its volume.

Solution :

Let radius of the sphere = r cm

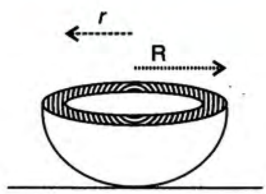
$$\Rightarrow \quad 4\pi r^2 = 616 \quad \Rightarrow \quad r^2 = \frac{616\times7}{4\times22} \quad \Rightarrow \quad r = 7 \text{ cm.}$$

$\therefore$ Its volume $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (7)^3 \text{ cm}^3 = \mathbf{1437\frac{1}{3}\ cm^3}$$ **Ans.**

14 The internal and external diameters of a hollow hemispherical vessel are 42 cm and 45·5 cm respectively. Find its capacity and also its outer curved surface area.

Solution :



Given; internal radius (r) $= \frac{42}{2}$ cm $= 21$ cm

And, external radius (R) $= \frac{45\cdot5}{2}$ cm $= 22\cdot75$ cm

Capacity of this vessel = Vol. of internal hemisphere

$$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (21)^3 \text{ cm}^3 = \mathbf{19{,}404\ cm^3}$$ **Ans.**

Outer curved surface area = Curved surface area of the outer hemisphere

$= 2\pi R^2$

$$= 2 \times \frac{22}{7} \times (22\cdot75)^2 \text{ cm}^2 = \mathbf{3253\cdot25\ cm^2}$$ **Ans.**

15 A solid spherical ball of iron with radius 6 cm is melted and recast into three solid spherical balls. The radii of the two of the balls are 3 cm and 4 cm respectively, determine the diameter of the third ball.

Solution :

Let the radius of the third ball = r cm

$\therefore$ Total volume of three balls formed = Volume of the ball melted

$\Rightarrow \quad \frac{4}{3}\pi(3)^3 + \frac{4}{3}\pi(4)^3 + \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3$

$\Rightarrow \quad 27 + 64 + r^3 = 216$

$\Rightarrow \quad r^3 = 125 \quad i.e. \quad r = 5$ cm

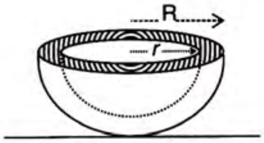
$\therefore$ **The diameter of the third ball** = 2 × 5 cm = **10 cm** **Ans.**

16 The internal and external diameters of a hollow hemispherical vessel are 21 cm and 25·2 cm respectively. Find the cost of painting it, all over, at the rate of ₹ 1.50 per cm^2.

Solution :

Let the internal and external radii be r and R.

$\therefore \quad r = \frac{21}{2}$ cm = 10·5 cm



and, $R = \frac{25 \cdot 2}{2}$ cm = 12·6 cm

Total surface area of the vessel

= External curved surface + Internal C.S.A. + Area of the ring formed at the top

$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= 2 \times \frac{22}{7} \times (12 \cdot 6)^2 \text{ cm}^2 + 2 \times \frac{22}{7} \times (10 \cdot 5)^2 \text{ cm}^2 + \frac{22}{7}(12 \cdot 6^2 - 10 \cdot 5^2) \text{ cm}^2$

$= 997 \cdot 92 \text{ cm}^2 + 693 \text{ cm}^2 + 152 \cdot 46 \text{ cm}^2 = 1843 \cdot 38 \text{ cm}^2$

$\Rightarrow$ **Cost of painting** = 1843·38 × ₹ 1.50

= **₹ 2,765.07** **Ans.**

EXERCISE 20(C)

1. The surface area of a sphere is 2464 cm^2, find its volume.
2. The volume of a sphere is 38808 cm^3; find its diameter and the surface area.
3. A spherical ball of lead has been melted and made into identical smaller balls with radius equal to half the radius of the original one. How many such balls can be made ?
4. How many balls each of radius 1 cm can be made by melting a bigger ball whose diameter is 8 cm ?
5. Eight metallic spheres; each of radius 2 mm, are melted and cast into a single sphere. Calculate the radius of the new sphere.
6. The volume of one sphere is 27 times that of another sphere. Calculate the ratio of their :

(i) radii,
(ii) surface areas.

7. If the number of square centimetres on the surface of a sphere is equal to the number of cubic centimetres in its volume, what is the diameter of the sphere ?
8. A solid metal sphere is cut through its centre into 2 equal parts. If the diameter of the sphere is $3\frac{1}{2}$ cm, find the total surface area of each part correct to two decimal places.
9. The internal and external diameters of a hollow hemispherical vessel are 21 cm and 28 cm respectively. Find :
   (i) internal curved surface area,
   (ii) external curved surface area,
   (iii) total surface area,
   (iv) volume of material of the vessel.
10. A solid sphere and a solid hemi-sphere have the same total surface area. Find the ratio between their volumes.
11. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted and recasted into a single solid sphere. Taking $\pi = 3{\cdot}1$, find the surface area of solid sphere formed.
12. The surface area of a solid sphere is increased by 21% without changing its shape. Find the percentage increase in its:
   (i) radius (ii) volume

20.7 Conversion of solids :

17 The radius of the base of a cone and the radius of a sphere are the same, each being 8 cm. Given that the volumes of these two solids are also the same, calculate the slant height of the cone.

Solution :

Given, volume of cone = volume of sphere

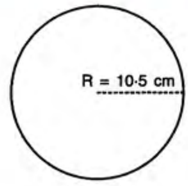
$$\therefore \quad \frac{1}{3}\pi (8)^2 h = \frac{4}{3}\pi (8)^3 \Rightarrow h = 32 \text{ cm}$$

$$\text{Now,} \quad l^2 = h^2 + r^2 \Rightarrow l^2 = 32^2 + 8^2 = 1088$$

$$\Rightarrow l = 32{\cdot}98 \text{ cm}$$ **Ans.**

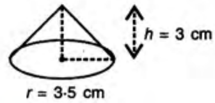
18 A metallic sphere of radius 10·5 cm is melted and then recast into small cones each of radius 3·5 cm and height 3 cm. Find the number of cones thus formed. [2005]

Solution :



$$\text{Volume of sphere melted} = \frac{4}{3}\pi R^3$$
$$= \frac{4}{3}\pi \times 10{\cdot}5 \times 10{\cdot}5 \times 10{\cdot}5 \text{ cm}^3$$

$$\text{Volume of each cone formed} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi \times 3{\cdot}5 \times 3{\cdot}5 \times 3 \text{ cm}^3$$



$$\therefore \textbf{No. of cones formed} = \frac{\text{Volume of the sphere melted}}{\text{Volume of each cone formed}}$$

$$= \frac{\frac{4}{3}\pi \times 10{\cdot}5 \times 10{\cdot}5 \times 10{\cdot}5}{\frac{1}{3}\pi \times 3{\cdot}5 \times 3{\cdot}5 \times 3} = 126$$ **Ans.**

19 A hollow metal sphere of internal and external radii 2 cm and 4 cm respectively is melted into a solid cone of base radius 4 cm. Find the height and slant height of the cone.

Solution :

According to the given statement :

Volume of metal in cone = Volume of metal in hollow sphere

$\Rightarrow \quad \frac{1}{3}\pi r^2 h$ for cone $= \frac{4}{3}\pi(R^3 - r^3)$ for hollow sphere

$\Rightarrow \quad \frac{1}{3} \times \pi \times 4^2 \times h = \frac{4}{3} \times \pi \times (4^3 - 2^3)$

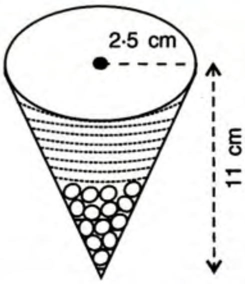
$\Rightarrow \quad 16h = 4 \times 56 \quad$ *i.e.* $\quad h = 14$ cm

$\because \; l^2 = h^2 + r^2 \Rightarrow l^2 = 14^2 + 4^2 = 212$ *i.e.* $l = \sqrt{212}$ cm $= 14{\cdot}56$ cm

$\therefore$ **Height = 14 cm** and **slant height = 14·56 cm** **Ans.**

20 A vessel is in the form of an inverted cone. Its height is 11 cm and the radius of its top, which is open, is 2·5 cm. It is filled with water up to the rim. When lead shots, each of which is a sphere of radius 0·25 cm, are dropped into the vessel, $\frac{2}{5}$ of the water flows out. Find the number of lead shots dropped into the vessel. [2003]

Solution :



Volume of water in the cone = Volume of the cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (2{\cdot}5)^2 \times 11 \text{ cm}^3$

Vol. of lead shots dropped = Vol. of water that overflows

$= \frac{2}{5} \times \frac{1}{3}\pi \times (2{\cdot}5)^2 \times 11 \text{ cm}^3$

Vol. of each lead shot $= \frac{4}{3}\pi(0{\cdot}25)^3 \text{ cm}^3$

$V = \frac{4}{3}\pi r^3$

$\therefore$ **The number of lead shots dropped** $= \dfrac{\text{Volume of lead shots dropped}}{\text{Volume of each lead shot}}$

$= \dfrac{\frac{2}{5} \times \frac{1}{3} \times \pi \times 2{\cdot}5 \times 2{\cdot}5 \times 11}{\frac{4}{3}\pi \times 0{\cdot}25 \times 0{\cdot}25 \times 0{\cdot}25} = $ **440** **Ans.**

EXERCISE 20(D)

1. A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2·5 cm and height 8 cm. Calculate the number of cones recast. **[2013]**

2. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone. **[2002]**

3. The radii of the internal and external surfaces of a metallic spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid right circular cone of height 32 cm. Find the diameter of the base of the cone.

4. Total volume of three identical cones is the same as that of a bigger cone whose height is 9 cm and diameter 40 cm. Find the radius of the base of each smaller cone, if height of each is 108 cm.

5. A solid rectangular block of metal 49 cm by 44 cm by 18 cm is melted and formed into a solid sphere. Calculate the radius of the sphere.

6. A hemispherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into conical shaped small containers each of diameter 3 cm and height 4 cm. How many containers are necessary to empty the bowl ?

7. A hemispherical bowl of *diameter* 7·2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4·8 cm. Find the height of the cone if it is completely filled. **[2010]**

8. A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0·5 cm. Find the number of spheres formed. **[2011]**

9. The total area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cones of radius 2·5 cm and height 8 cm. Calculate :
   (i) the radius of the solid sphere,
   (ii) the number of cones recast.
   Take $\pi = 3{\cdot}14$ **[2000]**

10. A solid metallic cone, with radius 6 cm and height 10 cm, is made of some heavy metal *A*. In order to reduce its weight, a conical hole is made in the cone as shown and it is completely filled with a lighter metal *B*. The conical hole has a diameter of 6 cm and depth 4 cm. Calculate the ratio of the volume of metal *A* to the volume of the metal *B* in the solid.

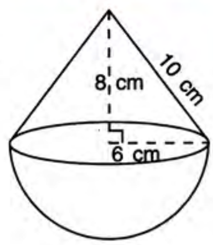
11. A hollow sphere of internal and external radii 6 cm and 8 cm respectively is melted and recast into small cones of base radius 2 cm and height 8 cm. Find the number of cones. **[2012]**

12. The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3·5 cm and height 7 cm. Calculate :
    (i) the radius of the sphere.
    (ii) the number of cones recast. (Take $\pi = \frac{22}{7}$) **[2014]**

20.8 Combination of Solids :

21 A toy is in the form of a cone mounted on a hemisphere with the same radius. The diameter of the base of the conical portion is 12 cm and its height is 8 cm. Determine the surface area and the volume of the toy (π = 3·14).

Solution :



Radius of the conical portion = Radius of the hemisphere

$= \frac{12}{2}$ cm = 6 cm.

And, height of the conical portion = 8 cm

$l^2 = h^2 + r^2 \Rightarrow l^2 = 8^2 + 6^2 = 100 \Rightarrow l = 10$ cm

$\Rightarrow$ The slant height l = 10 cm.

Now, the surface area of the toy = Curved surface area of the conical portion + curved surface area of the hemisphere

$= \pi rl + 2\pi r^2$

$= 3{\cdot}14(6 \times 10 + 2 \times 6^2)\ cm^2$

$= \mathbf{414{\cdot}48\ cm^2}$ **Ans.**

Also, the volume of the toy = Vol. of the cone + vol. of hemisphere

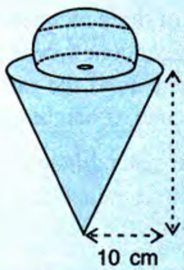
$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi(\frac{1}{3} \times 6^2 \times 8 + \frac{2}{3} \times 6^3)\ cm^3$

$= 3{\cdot}14(96 + 144)\ cm^3$

$= \mathbf{753{\cdot}6\ cm^3}$ **Ans.**

22 The given block is made of two solids : a cone and a hemisphere. If the height and the base-radius of the cone are 24 cm and 10 cm respectively and the diameter of the hemisphere is 10 cm; find the total surface area of the block. (Take $\pi = 3{\cdot}14$)



Solution :

Let radius of the cone be denoted by R, therefore R = 10 cm,

its slant-height $(l) = \sqrt{h^2 + R^2} = \sqrt{24^2 + 10^2}$ cm = 26 cm

Also, let radius of the hemisphere be denoted by r,

therefore, $r = \frac{10}{2}$ cm = 5 cm

Total surface area of the block = Total surface area of the cone – area of the base of the cone which is in contact with the hemisphere + curved surface area of the hemisphere

$= (\pi Rl + \pi R^2) - \pi r^2 + \frac{4\pi r^2}{2}$

$= \pi\ [Rl + R^2 - r^2 + 2r^2]$

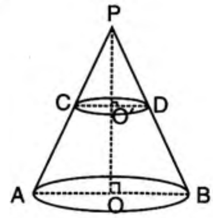
$= 3{\cdot}14\ (10 \times 26 + 10^2 + 5^2)$ cm $\quad [2r^2 - r^2 = r^2]$

$= \mathbf{1208{\cdot}9\ cm^2}$ **Ans.**

23 The height of a solid cone is 30 cm. A small cone is cut off from the top of it such that the base of the cone cut off and the base of the given cone are parallel to each other. If the volume of the cone cut and the volume of the original cone are in the ratio 1 : 27; find the height of the the remaining part of the given cone.

Solution :

According to the given statement, the figure will be as drawn alongside. Clearly, CD is parallel to AB.



PO = 30 cm = height (H) of the given cone

PO′ = h cm = height of the cone cut.

OB = R cm = radius of the given cone

and, O′D = r cm = radius of the cone cut.

It can easily be shown that Δ POB and Δ PO′D are similar and so $\frac{PO}{PO'} = \frac{OB}{O'D}$

$$\Rightarrow \quad \frac{H}{h} = \frac{R}{r} \qquad \text{...... I}$$

Given : $\frac{\text{Volume of cone cut}}{\text{Volume of given cone}} = \frac{1}{27} \quad \Rightarrow \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi R^2 H} = \frac{1}{27}$

i.e., $\left(\frac{r}{R}\right)^2 \times \frac{h}{H} = \frac{1}{27}$

$\Rightarrow \left(\frac{h}{H}\right)^2 \times \frac{h}{H} = \frac{1}{27}$ [From equation I, $\frac{r}{R} = \frac{h}{H}$]

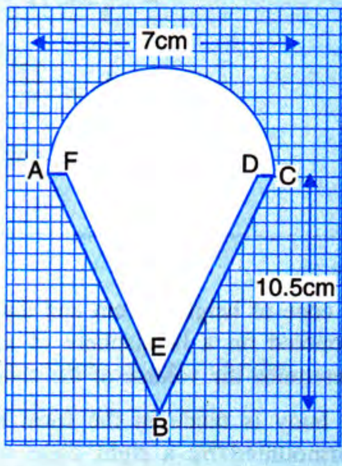
$\Rightarrow \frac{h^3}{H^3} = \frac{1}{27}$ and $\frac{h}{H} = \frac{1}{3}$

$\Rightarrow \frac{h}{30} = \frac{1}{3}$ and $h = \frac{1}{3} \times 30$ cm = 10 cm

∴ **Height of the remaining part of the cone** = PO – PO′

= H – h = (30 – 10) cm = **20 cm** **Ans.**

24 The given figure shows the cross-section of an ice-cream cone consisting of a cone surmounted by a hemisphere. The radius of the hemisphere is 3·5 cm and the height of the cone is 10·5 cm. The outer shell ABCDEF is shaded and is not filled with ice-cream. AF = DC = 0·5 cm, AB // FE and BC // ED.



Calculate :

(i) the volume of the ice-cream in the cone (the internal volume of the cone including the hemisphere) in cm^3;

(ii) the volume of the outer shell (the shaded portion) in cm^3.

In each case, give your answer correct to the nearest cm^3.

Solution :

Given, diameter of the outer conical shell = AC = 7 cm and, diameter of the inner conical shell = AC − AF − DC

$$= 7\text{ cm} - 0{\cdot}5\text{ cm} - 0{\cdot}5\text{ cm} = 6\text{ cm}$$

∴ Radius (R) of the outer conical shell $= \frac{7}{2}$ cm = 3·5 cm

and, radius (r) of the inner conical shell $= \frac{6}{2}$ cm = 3 cm

Also, given that the height of the outer conical shell = 10·5 cm

It can easily be shown that outer-face ABC and inner-face FED of the given conical shell form two similar triangles.

$$\therefore \quad \frac{\text{Height of the inner conical shell}}{\text{Height of the outer conical shell}} = \frac{\text{Diameter of the inner - shell}}{\text{Diameter of the outer - shell}}$$

$$\Rightarrow \quad \frac{h}{H} = \frac{2r}{2R}$$

i.e.

$$h = \frac{r}{R} \times H = \frac{3}{3{\cdot}5} \times 10{\cdot}5\text{ cm} = 9\text{ cm}$$

(i) **Required volume of ice-cream**

= Internal volume of the cone + Volume of the hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi R^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 9\text{ cm}^3 + \frac{2}{3} \times \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5 \times 3{\cdot}5\text{ cm}^3$$

= 174·69 cm^3 = **175 cm^3** (correct to the nearest cm^3) **Ans.**

(ii) **Volume of the conical-shell**

= Its external volume − Its internal volume

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7}\,[3{\cdot}5^2 \times 10{\cdot}5 - 3^2 \times 9]\text{ cm}^3$$

= 49·89 cm^3 = **50 cm^3** **Ans.**

EXERCISE 20(E)

1. A cone of height 15 cm and diameter 7 cm is mounted on a hemisphere of same diameter. Determine the volume of the solid thus formed.
2. A buoy is made in the form of hemisphere surmounted by a right cone whose circular base coincides with the plane surface of hemisphere. The radius of the base of the cone is 3·5 metres and its volume is two-third of the hemisphere. Calculate the height of the cone and the surface area of the buoy, correct to two places of decimal.
3. From a rectangular solid of metal 42 cm by 30 cm by 20 cm, a conical cavity of diameter 14 cm and depth 24 cm is drilled out. Find :
   (i) the surface area of remaining solid,
   (ii) the volume of remaining solid,
   (iii) the weight of the material drilled out if it weighs 7 gm per cm^3.
4. A cubical block of side 7 cm is surmounted by a hemisphere of the largest size. Find the surface area of the resulting solid.

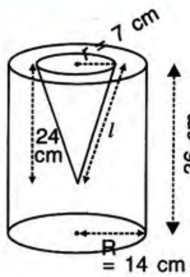
5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the rim. When lead shots, each of which is a sphere of radius 0·5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.
6. A hemi-spherical bowl has negligible thickness and the length of its circumference is 198 cm. Find the capacity of the bowl.
7. Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r cm.
8. The radii of the bases of two solid right circular cones of same height are r_1 and r_2 respectively. The cones are melted and recast into a solid sphere of radius R. Find the height of each cone in terms of r_1, r_2 and R.
9. A solid metallic hemisphere of diameter 28 cm is melted and recast into a number of identical solid cones, each of diameter 14 cm and height 8 cm. Find the number of cones so formed.
10. A cone and a hemisphere have the same base and the same height. Find the ratio between their volumes.

20.9 Miscellaneous Problems :

25 From a solid cylinder of height 36 cm and radius 14 cm, a conical cavity of radius 7 cm and height 24 cm is drilled out. Find the volume and the total surface area of the remaining solid.

Solution :

According to the given statement, the figure will be as shown alongside :



Clearly, for solid cylinder, height (H) = 36 cm and radius (R) = 14 cm. And for conical cavity, height (h) = 24 cm, radius (r) = 7 cm and slant height (l) = $\sqrt{h^2+r^2}$ = $\sqrt{24^2+7^2}$ cm = 25 cm

Volume of the remaining solid

= Volume of cylinder – volume of conical cavity

$= \pi R^2 H - \frac{1}{3}\pi r^2 h$

$= \frac{22}{7} \times (14)^2 \times 36 \text{ cm}^3 - \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24 \text{ cm}^3$

$= 22176 \text{ cm}^3 - 1232 \text{ cm}^3 = \mathbf{20944 \text{ cm}^3}$ **Ans.**

Total surface area of the remaining solid

= Area of the base of the cylinder + curved surface area of the cylinder + area of the top ring + curved surface area of the conical cavity

$= \pi R^2 + 2\pi RH + \pi(R^2 - r^2) + \pi rl$

$= \frac{22}{7}[14^2 + 2 \times 14 \times 36 + (14^2 - 7^2) + 7 \times 25]\text{cm}^2$

$= \frac{22}{7} \times 1526 \text{ cm}^2 = \mathbf{4796 \text{ cm}^2}$ **Ans.**

26 A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find :

(i) the radius and

(ii) the slant height of the heap. Give your answer correct to one place of decimal. **[2004]**

Solution :

(i) Volume of conical heap = Volume of bucket

$\Rightarrow \quad \frac{1}{3}\pi r^2 \times 24 = \pi(18)^2 \times 32$

$\Rightarrow \quad r^2 = 1296 \quad$ *i.e.* $r =$ **36 cm** **Ans.**

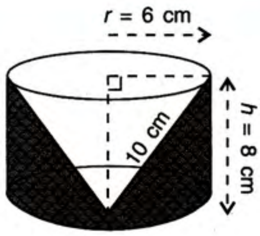
(ii) $l^2 = 24^2 + 36^2 \quad$ [Since, $l^2 = h^2 + r^2$]

$\Rightarrow \quad l^2 = 576 + 1296$

$= 1872 \quad \therefore$ $l =$ **43·3 cm** **Ans.**

27 From a solid cylinder, whose height is 8 cm and radius is 6 cm, a conical cavity of height 8 cm and of base radius 6 cm is hollowed out. Find the volume of the remaining solid. Also, find the total surface area of the remaining solid.

Solution :



For the conical cavity, radius $r = 6$ cm and height $h = 8$ cm

$\Rightarrow$ Slant height, $l = \sqrt{h^2 + r^2} = \sqrt{8^2 + 6^2}$ cm = 10 cm

The volume of the remaining solid

= Volume of cylinder – volume of cone

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$

$= \frac{22}{7}(6^2 \times 8 - \frac{1}{3} \times 6^2 \times 8)$ cm^3 = **603 $\frac{3}{7}$ cm^3** **Ans.**

The total surface area of the remaining solid

= Curved surface of cylinder + curved surface of the cone + base of the cylinder

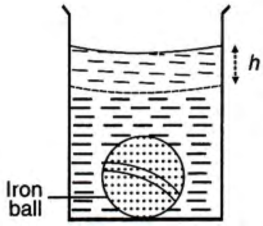
$= 2\pi rh + \pi rl + \pi r^2$

$= \frac{22}{7}(2 \times 6 \times 8 + 6 \times 10 + 6^2)$ cm^2 = **603 $\frac{3}{7}$ cm^2 Ans.**

28 A cylindrical beaker, whose base has a radius of 15 cm, is filled with water up to a height of 20 cm. A heavy iron spherical ball of radius 10 cm is dropped to submerge completely in water in the beaker. Find the increase in the level of water.

Solution :

Let the increase in the level of water be h cm.



$\Rightarrow$ Volume of water in beaker that rises up = $\pi(15)^2\ h\ \text{cm}^3$

and volume of spherical ball = $\frac{4}{3}\pi(10)^3\ \text{cm}^3$

Clearly, volume of water that rises up = volume of ball

$\therefore \quad \pi\,(15)^2\,h = \frac{4}{3}\pi(10)^3$ and, $h = 5\frac{25}{27}$ cm **Ans.**

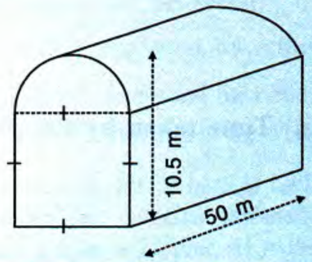
(b) Based on uniform cross-section :

When a body has uniform cross-section, its :

1. Volume = Area of cross-section × length (or, height).
2. Surface area (excluding cross-section)
   = Perimeter of cross-section × length (or, height).

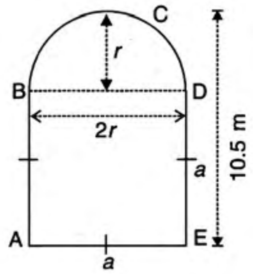
29 The cross-section of a railway tunnel is a square surmounted by a semi-circle as shown in the figure.

The tunnel is 50 m long. Find the cost of plastering the internal surface of the tunnel (excluding the floor) at the rate of ₹ 10 per m^2.



Solution :

Let each side of the square be 'a' metre.



$\therefore \quad a + r = 10{\cdot}5$

or, $\quad a + \frac{a}{2} = 10{\cdot}5 \qquad$ [Since, $a = 2r$]

$\Rightarrow \quad a = 7$ m and $r = \frac{7}{2}$ m.

$\therefore$ Perimeter of cross-section (excluding the floor)

$= AB + \text{Semi-circle } BCD + DE$

$= a + \frac{2\pi r}{2} + a$

$= (7 + \frac{22}{7} \times \frac{7}{2} + 7)\ \text{m} = 25\ \text{m}$

Internal surface area (excluding the floor) = Perimeter of cross-section × length

$= 25 \times 50\ \text{m}^2 = 1250\ \text{m}^2$

Cost of plastering = ₹ 10 × 1250 = **₹ 12,500** **Ans.**

30 Water flows through a cylindrical pipe of internal diameter 7 cm at 5 metre per second. Calculate :

(i) the volume, in litres, of water discharged by the pipe in one minute,

(ii) the time, in minutes, the pipe would take to fill an empty rectangular tank 4m × 3m × 2·31m.

Solution :

> When water flows through a pipe of uniform cross-section A, with speed V, the volume of water that flows through the pipe in unit time = A × V.

Given that the internal diameter of the cylindrical pipe = 7 cm

so, its radius = $\frac{7}{2}$ cm

(i) Area of internal cross-section of the pipe $= \pi r^2$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2 = 38{\cdot}5 \text{ cm}^2$$

Since, speed of water through it $= 5 \text{ ms}^{-1} = 500 \text{ cms}^{-1}$

∴ Volume of water discharged by the pipe in 1 sec

$$= 38{\cdot}5 \times 500 \text{ cm}^3$$
$$= 19{,}250 \text{ cm}^3$$

⇒ **Volume of water discharged in 1 minute (60 sec)**

$$= 19{,}250 \times 60 \text{ cm}^3$$
$$= \frac{19{,}250 \times 60}{1{,}000} \text{ litre} = \textbf{1,155 litre} \qquad \textbf{Ans.}$$

(ii) **Time taken by the pipe to fill the given tank**

$$= \frac{\text{Volume of the tank}}{\text{Volume of water discharged by the pipe in unit time}}$$

$$= \frac{400 \text{ cm} \times 300 \text{ cm} \times 231 \text{ cm}}{19{,}250 \text{ cm}^3 \text{ per second}}$$

$$= 1440 \text{ second} = \frac{1440}{60} \text{ minute} = \textbf{24 minute} \qquad \textbf{Ans.}$$

EXERCISE 20(F)

1. From a solid right circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed. Find the volume of the remaining solid.
2. From a solid cylinder whose height is 16 cm and radius is 12 cm, a conical cavity of height 8 cm and of base radius 6 cm is hollowed out. Find the volume and total surface area of the remaining solid.
3. A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 80 m, calculate the total area of canvas required. Also, find the total cost of canvas used at ₹ 15 per metre if the width is 1·5 m.
4. A circus tent is cylindrical to a height of 8 m surmounted by a conical part. If total height of the tent is 13 m and the diameter of its base is 24 m; calculate :
   (i) total surface area of the tent,
   (ii) area of canvas, required to make this tent allowing 10% of the canvas used for folds and stitching.
5. A cylindrical boiler, 2 m high, is 3·5 m in diameter. It has a hemispherical lid. Find the volume of its interior, including the part covered by the lid.
6. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylindrical part is $4\frac{2}{3}$ m and the diameter of hemisphere is 3·5 m. Calculate the capacity and the internal surface area of the vessel.

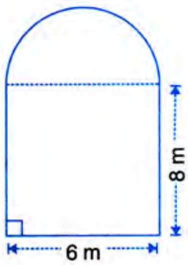
7. A wooden toy is in the shape of a cone mounted on a cylinder as shown alongside.

If the height of the cone is 24 cm, the total height of the toy is 60 cm and the radius of the base of the cone = twice the radius of the base of the cylinder = 10 cm; find the total surface area of the toy. [Take $\pi = 3{\cdot}14$]

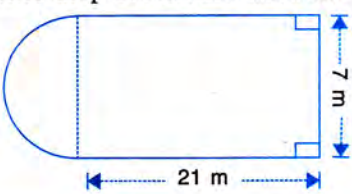
8. A cylindrical container with diameter of base 42 cm contains sufficient water to submerge a rectangular solid of iron with dimensions 22 cm × 14 cm × 10·5 cm. Find the rise in level of the water when the solid is submerged.

9. Spherical marbles of diameter 1·4 cm are dropped into beaker containing some water and are fully submerged. The diameter of the beaker is 7 cm. Find how many marbles have been dropped in it if the water rises by 5·6 cm ?

10. The cross-section of a railway tunnel is a rectangle 6 m broad and 8 m high surmounted by a semi-circle as shown in the figure. The tunnel is 35 m long. Find the cost of plastering the internal surface of the tunnel (excluding the floor) at the rate of ₹ 2.25 per m^2.

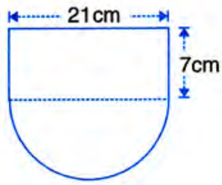


11. The horizontal cross-section of a water tank is in the shape of a rectangle with semi-circle at one end, as shown in the following figure. The water is 2·4 metres deep in the tank. Calculate the volume of water in the tank in gallons.



Given : 1 gallon = 4·5 litres

12. The given figure shows the cross-section of a water channel consisting of a rectangle and a semi-circle. Assuming that the channel is always full, find the volume of water discharged through it in one minute if water is flowing at the rate of 20 cm per second. Give your answer in cubic metres correct to one place of decimal.



13. An open cylindrical vessel of internal diameter 7 cm and height 8 cm stands on a horizontal table. Inside this is placed a solid metallic right circular cone, the diameter of whose base is $3\frac{1}{2}$ cm and height 8 cm. Find the volume of water required to fill the vessel.

If this cone is replaced by another cone, whose height is $1\frac{3}{4}$ cm and the radius of whose base is 2 cm, find the drop in the water level.

14. A cylindrical can, whose base is horizontal and of radius 3·5 cm, contains sufficient water so that when a sphere is placed in the can, the water just covers the sphere. Given that the sphere just fits into the can, calculate :

(i) the total surface area of the can in contact with water when the sphere is in it;

(ii) the depth of water in the can before the sphere was put into the can.

15. A hollow cylinder has solid hemisphere inward at one end and on the other end it is closed with a flat circular plate. The height of water is 10 cm when flat circular surface is downward. Find the level of water, when it is inverted upside down, common diameter is 7 cm and height of the cylinder is 20 cm.

EXERCISE 20(G)

1. What is the least number of solid metallic spheres, each of 6 cm diameter, that should be melted and recast to form a solid metal cone whose height is 45 cm and diameter 12 cm ?

2. A largest sphere is to be carved out of a right circular cylinder of radius 7 cm and height 14 cm. Find the volume of the sphere. (Answer correct to the nearest integer).

3. A right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in identical cones of height 12 cm and diameter 6 cm having a hemi-spherical shape on the top. Find the number of cones required.

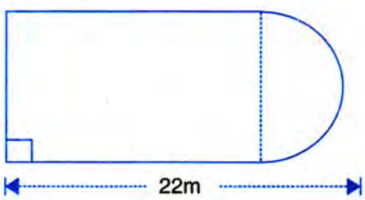
4. A solid is in the form of a cone standing on a hemi-sphere with both their radii being equal to 8 cm and the height of cone is equal to its radius. Find, in terms of π, the volume of the solid.

5. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0·2 cm. Find the length of the wire.

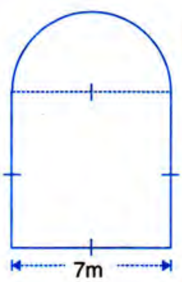
6. Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube.

7. An iron pole consisting of a cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that 1 cm^3 of iron has 8 gm of mass (approx). (Take $\pi = \frac{355}{113}$)

8. In the following diagram a rectangular platform with a semi-circular end on one side is 22 metres long from one end to the other end. If the length of the half circumference is 11 metres, find the cost of constructing the platform, 1·5 metres high at the rate of ₹ 4 per cubic metres.



9. The cross-section of a tunnel is a square of side 7 m surmounted by a semi-circle as shown in the adjoining figure. The tunnel is 80 m long.



Calculate :

(i) its volume,

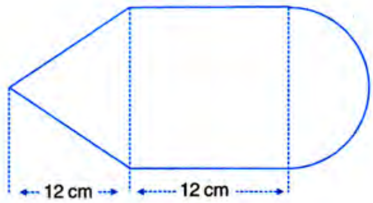
(ii) the surface area of the tunnel (excluding the floor) and

(iii) its floor area.

10. A cylindrical water tank of diameter 2·8 m and height 4·2 m is being fed by a pipe of diameter 7 cm through which water flows at the rate of 4 m s^{-1}. Calculate, in minutes, the time it takes to fill the tank.

11. Water flows, at 9 km per hour, through a cylindrical pipe of cross-sectional area 25 cm^2. If this water is collected into a rectangular cistern of dimensions 7·5 m by 5 m by 4 m; calculate the rise in level in the cistern in 1 hour 15 minutes.

12. The given figure shows the cross-section of a cone, a cylinder and a hemisphere all with the same diameter 10 cm, and the other dimensions are as shown.



Calculate :

(a) the total surface area,

(b) the total volume of the solid and

(c) the density of the material if its total weight is 1·7 kg.

13. A solid, consisting of a right circular cone standing on a hemisphere, is placed upright in a right circular cylinder, full of water, and touches the bottom. Find the volume of water left in the cylinder, having given that the radius of the cylinder is 3 cm and its height is 6 cm; the radius of the hemisphere is 2 cm and the height of cone is 4 cm. Give your answer to the nearest cubic centimetre.

14. A metal container in the form of a cylinder is surmounted by a hemisphere of the same radius. The internal height of the cylinder is 7 m and the internal radius is 3·5 m. Calculate :

(i) the total area of the internal surface, excluding the base;

(ii) the internal volume of the container in m^3.

15. An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and height of the cylindrical part is 50 m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for fold and for stitching. Give your answer to the nearest m^2. **[2001]**

16. A test tube consists of a hemisphere and a cylinder of the same radius. The volume of the water required to fill the whole tube is $\frac{5159}{6}$ cm^3, and $\frac{4235}{6}$ cm^3 of water is required to fill the tube to a level which is

4 cm below the top of the tube. Find the radius of the tube and the length of its cylindrical part.

17. A solid is in the form of a right circular cone mounted on a hemisphere. The diameter of the base of the cone, which exactly coincides with hemisphere, is 7 cm and its height is 8 cm. The solid is placed in a cylindrical vessel of internal radius 7 cm and height 10 cm. How much water, in cm^3, will be required to fill the vessel completely ?

18. Two solid spheres of radii 2 cm and 4 cm are melted and recast into a cone of height 8 cm. Find the radius of the cone so formed. **[2015]**