25 Probability

25.1 Introduction :

Consciously or unconsciously, all of us sometime use the phrases like : **'most likely'**, **'almost uncertain'**, **'most probably'**, **'no chance at all'**, etc.

If we read these phrases carefully, we will find that all of them involve an element of uncertainty. The measure of uncertainty is called the **theory of probability**.

The study of the theory of probabilities is of great mathematical interest and of great practical importance.

25.2 Some Basic Terms and Concepts :

1. Experiment :

A process which results in some well-defined outcome is known as an **experiment**.

For example :

(i) When a coin is tossed, we shall be getting either a head or a tail *i.e.* its outcome is a head or a tail, *which is well-defined.*

(ii) When a die is thrown the possible outcomes are 1, 2, 3, 4, 5 and 6, *which are also well-defined.*

2. Random Experiment :

Random experiment means all the outcomes of the experiment are known in advance, but any specific outcome of the experiment is not known in advance.

For example :

(i) Tossing a coin is a random experiment because there are only two possible outcomes, head and tail, and these outcomes are known well in advance. But the specific outcome of the experiment *i.e.* whether a head or a tail is not known in advance.

(ii) Throwing a die is a random experiment because we know in advance that there are only six possible outcomes of the experiment *i.e.* 1, 2, 3, 4, 5 and 6. But it is not possible to know which of these six numbers will finally be the result.

A random experiment may result in two or more outcomes; for example :
(i) tossing a coin. (ii) throwing a dice, etc.

3. Sample Space :

The set of all possible outcomes of an experiment is called ***sample space*** and is, in general, denoted by letter **S**.

For example :

(i) When we toss a coin once, it may come up in either of two ways : Head (H) or Tail (T). So, there are two possible outcomes of this random experiment. Thus the sample space (S) of this random experiment is given by S = {H, T}

(ii) When we roll a dice once, it may land with any of its 6 faces pointing upward. Thus, the outcome of this experiment is getting any of the six numbers 1, 2, 3, 4, 5 and 6. Hence, the sample space for the experiment is S = {1, 2, 3, 4, 5, 6}.

(iii) When two coins are tossed together, the random experiment may result :

(a) head (H) on the first coin and head (H) on the second coin.

(b) head (H) on the first coin and tail (T) on the second coin.

(c) tail (T) on the first coin and head (H) on the second coin.

(d) tail (T) on the first coin and tail (T) on the second coin.

Thus the corresponding sample space S = {(H, H), (H, T), (T, H), (T, T)}.

(iv) When two dice are rolled together, the corresponding sample space for the random experiment is as given below :

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

(v) When a coin and a dice are tossed together, the corresponding sample space for the random experiment is as given below :

S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6),
(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}.

4. Equally Likely Outcomes :

In case of tossing a coin :

(i) It is known, in advance, that the coin will land with its head or tail up.

(ii) It is reasonably assumed that each outcome, a head or a tail, is as likely to occur as the other. In other words, we say that there are equal chances for the coin to land with its head or tail up.

Referring to the terms used in this chapter, we say that the outcomes, head and tail, are **equally likely**.

In case of throwing a dice :

(i) It is known, in advance, that the dice will show the number 1, 2, 3, 4, 5 or 6 on the upper-most face.

(ii) It can reasonably be accepted that each of the numbers 1, 2, 3, 4, 5 and 6 has the same possibility to come to the upper-most face.

Hence, showing up the numbers 1, 2, 3, 4, 5 and 6 on the throwing of a die are **equally likely outcomes**.

Are the outcomes of all experiments equally likely ?

Suppose a bag contains 6 red and 2 yellow balls. Let a ball be drawn from the bag without looking into it. The ball that will come out will either be a red or a yellow ball.

Are the outcomes, a red ball and a yellow ball, equally likely ?

No.

Reason :

Since the bag contains 6 red and 2 yellow balls, then in a single draw of a ball from this bag (without looking into it); it is more likely to get a red ball than a yellow ball. Hence, the outcomes are **not equally likely**.

However, if the bag contains equal numbers of red balls and yellow balls, the outcomes are equally likely.

5. An Event :

An outcome of a random experiment is called an event. In other words, an event is something that happens.

On tossing a coin, the possible outcome is a head (H) or a tail (T). Here, getting a head or a tail is an event of the experiment of throwing a coin. Similarly, in throwing a cubical dice the six possible outcomes are 1, 2, 3, 4, 5 or 6. Thus, getting 1, 2, 3, 4, 5 or 6 on the upper face of the dice is an event of the experiment of throwing a dice.

In the same way, if a card is drawn from a well-shuffled pack of 52 playing cards, any one of them can be the outcome. So, there are 52 events of the random experiment of drawing a card from a pack of 52 playing cards.

25.3 Measurement of Probability :

The probability of an event denotes the likelihood of its happening.

If in a random experiment, the total number of events (outcomes) are n out of which m events (outcomes) are favourable to a particular event E; then the probability of happening of event E is denoted by **P(E)** and is equal to the ratio $\frac{m}{n}$.

i.e. **P(E)** = Probability of the happening of event E

$$= \frac{m}{n}$$

$$= \frac{\text{Number of events (outcomes) favourable to E}}{\text{Total number of all possible outcomes}}$$

For example :

If a dice is rolled once, and an even number is required on the upper face of it; then in this experiment :

Total number of outcomes = 6 (any of 1, 2, 3, 4, 5 and 6) and, number of favourable outcomes = 3 (any of 2, 4 and 6)

$\therefore$ **Probability of getting an even number on the upper face**

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{3}{6} = \frac{1}{2}$$

1. Empirical (or, experimental) Probability :

When the probability is based on an actual experiment, it is called an **empirical** (or, experimental) **probability**.

For example :

If a coin is tossed 100 times and the outcomes of this experiment are 57 heads and 43 tails, the probability of a head is $\frac{57}{100}$ and that of a tail is $\frac{43}{100}$. Since these probabilities are based on the actual experiment of tossing a coin 100 times; they are experimental (or, empirical) probabilities.

(i) For finding the experimental probability, an adequate recording of the outcomes is required.

(ii) Experimental probabilities are only 'estimates'. If the same experiment of tossing a coin 100 times is performed again, it will not necessarily give the same results of getting the number of heads and the number of tails. And, so the probabilities for a head and a tail will also not be the same.

2. Classical (or, theoretical) Probability :

When a repetition of an experiment can be avoided for calculating the exact probability, the probability so obtained is called **classical** (or, theoretical) probability.

(i) The empirical probability can be applied to every event associated with an experiment which can be repeated a large number of times.

(ii) In theoretical (classical) probability, we make certain assumptions and one of these assumptions is that the outcomes are equally likely.

(iii) Probability of an event (outcome) $= \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$

(iv) In this chapter, the probability means theoretical (or, classical) probability.

1 Find the probability of getting a head when a coin is tossed once.

Solution :

In the random experiment of tossing a coin once, the total number of possible outcomes is 2 which are Head (H) and Tail (T).

Favourable outcome is **'getting a head'**.

$\Rightarrow$ Number of favourable outcome = 1

$\therefore$ **P(getting a head)** $= \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}} = \frac{1}{2}$ **Ans.**

An event, having only one favourable outcome, is called an elementary event, as shown in the example given above. Similarly, events in the following examples are also elementary events.

2 A bag contains a black ball, a red ball and a green ball, all the balls are identical in shape and size. Mohit takes out a ball from the bag, without looking into it. What is the probability that the ball drawn is :

(i) red ball ? (ii) black ball ? (iii) green ball ?

Solution :

When Mohit takes out a ball without looking into the bag, the outcomes of the experiment are equally likely.

Clearly, the total number of possible outcomes = 3

(i) The number of favourable outcome (getting a red ball) = 1

⇒ **The probability of drawing a red ball**

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{3}$$ **Ans.**

In short; **P(red ball)** $= \frac{1}{3}$ **Ans.**

Similarly, (ii) **P(drawing a black ball)** $= \frac{1}{3}$ **Ans.**

and, (iii) **P(getting a green ball)** $= \frac{1}{3}$ **Ans.**

The sum of the probabilities of all the elementary events of an experiment is always one. In the example, given above :

P(red ball) + P(black ball) + P(green ball) $= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

3 In a single throw of a die, find the probability of getting a number :
(i) greater than 2 (ii) less than or equal to 2 (iii) not greater than 2.

Solution :

In a single throw of a die, the total possible outcomes are 6 (1, 2, 3, 4, 5 and 6).

(i) Out of 1, 2, 3, 4, 5 and 6; the numbers greater than 2 are 3, 4, 5 and 6.

∴ Total number of favourable outcomes = 4 (3, 4, 5 and 6)

⇒ **P(getting a number greater than 2)**

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}} = \frac{4}{6} = \frac{2}{3}$$ **Ans.**

(ii) Out of all possible outcomes 1, 2, 3, 4, 5 and 6, the numbers less than or equal to 2 are 1 and 2.

∴ Total number of favourable outcomes = 2

⇒ **P(getting a number less than or equal to 2)** $= \frac{2}{6} = \frac{1}{3}$ **Ans.**

(iii) Out of all possible outcomes 1, 2, 3, 4, 5 and 6, the numbers not greater than 2 are 1 and 2 only.

∴ The number of favourable outcomes = 2

⇒ **P(getting a number not greater than 2)** $= \frac{2}{6} = \frac{1}{3}$ **Ans.**

1. In a single throw of a die; getting a number less than or equal to 2 and getting a number not greater than 2 mean the same.
   For this reason : P(getting a number less than or equal to 2)
   = P(getting a number not greater than 2)
2. If the event of getting number greater than 2 is denoted by E.
   Then the event of getting a number not greater than 2 (or, a number less than or equal to 2) is denoted by **not E** or by $\overline{E}$.

Thus, $P(E) = P(\text{getting number greater than 2}) = \frac{2}{3}$

And, $P(\bar{E}) = P(\text{not E}) = P(\text{getting number not greater than 2}) = \frac{1}{3}$

$\therefore \quad \mathbf{P(E) + P(\bar{E}) = \frac{2}{3} + \frac{1}{3} = 1}$

Also, $P(E) + P(\bar{E}) = 1 \Rightarrow \mathbf{P(\bar{E}) = 1 - P(E)}$

i.e. $\mathbf{P(\text{not E}) = 1 - P(E)}$

3. **E and $\bar{E}$** (not E) are called **complementary events** *i.e.*, for any event E, the event of non-occurence of E is called its complementary event and is denoted by $\bar{E}$.
4. The sum of probabilities of an event and its complementary event is always 1.

4 From a well-shuffled deck of 52 cards, one card is drawn. Find the probability that the card drawn will : (i) be a face card (ii) not be a face card.

Solution :

1. A deck of playing-cards consists of 52 cards which are divided into 4 suits of 13 cards each.
2. The four suits in a deck of cards are : spades (♠), hearts (♥), diamonds (♦) and clubs (♣). Thus a deck consists of 13 cards of spades, 13 cards of hearts, 13 cards of diamonds and 13 cards of clubs.
3. The 13 cards of each suit are : an ace, a king, a queen, a jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2.
4. Kings, queens and jacks are called face cards. A deck of 52 cards contains 4-kings, 4-queens and 4-jacks.
   $\therefore$ Total number of face cards in a deck of 52 cards is : $4 + 4 + 4 = 12$.
5. Clubs and spades are of black colour whereas hearts and diamonds are of red colour.

Since one card is drawn from a well-shuffled deck of 52 cards, the total number of all possible outcomes = 52.

(i) A deck of 52 cards contains $4 \times 3 = 12$ face cards

$\Rightarrow$ The number of favourable outcomes = 12

$\therefore$ **P(card drawn will be a face card)** $= \frac{12}{52} = \frac{3}{13}$ **Ans.**

(ii) Since total number of cards in a deck = 52 and number of face cards = 12

$\therefore$ The number of cards which are not face cards $= 52 - 12 = 40$

$\Rightarrow$ The total number of outcomes = 52 and
the total number of favourable outcomes = 40

$\therefore$ **P(card drawn is not a face card)** $= \frac{40}{52} = \frac{10}{13}$ **Ans.**

Alternative Method :

If E represents that the card drawn is a face-card, then the card drawn is not a face-card is represented by $\overline{E}$ or **'not E'**.

Since, P(E) = Probability that the card drawn is a face-card. = $\frac{3}{13}$

And, P(E) + P(not E) = 1

$\Rightarrow$ **P(not E)** = $1 - \frac{3}{13} = \frac{\mathbf{10}}{\mathbf{13}}$

i.e. **P(card drawn is not a face card)** = $\frac{\mathbf{10}}{\mathbf{13}}$ **Ans.**

5 In a badminton match between Rajesh and Joseph, the probability of winning of Rajesh is 0·58. Find the probability of :
(i) not winning of Rajesh,
(ii) winning of Joseph.

Solution :

(i) $\because$ P(winning of Rajesh) + P(not winning of Rajesh) = 1

$\Rightarrow$ 0·58 + P(not winning of Rajesh) = 1

$\Rightarrow$ **P(not winning of Rajesh)** = 1 − 0·58 = **0·42** **Ans.**

(ii) **P(winning of Joseph)** = P(not winning of Rajesh) = **0·42** **Ans.**

6 In a single throw of a die, find the probability of getting :
(i) 7 (ii) a number less than 7.

Solution :

(i) When a die is thrown, the possible outcomes are 1, 2, 3, 4, 5 and 6.

Since no face of the die has the number 7 marked on it, there is no outcome favourable to 7.

$\Rightarrow$ The number of favourable outcomes = 0

$\Rightarrow$ **P(getting a number 7)**

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{0}{6} = \mathbf{0}$$ **Ans.**

(ii) Since every face of a die is marked with a number less than 7.

$\therefore$ The number of favourable outcomes = the number of faces = 6

$\Rightarrow$ **P(getting a number less than 7)**

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{6}{6} = \mathbf{1}$$ **Ans.**

Impossible Event and Sure Event

1. If the probability of an event = 0; the event is called an **impossible event.**
In part (i) of the example, given above, it is impossible to get a number 7 in a single throw of a die, so the probability of this event is 0.

2. If the probability of an event = 1; the event is called a **certain event** or a **sure event.**

   In part (ii) of the example, given above, it is sure that a number less than 7 will be obtained whenever a die is thrown, so the probability of the event is 1.

3. **Probability of any event can never be less than 0 or more than 1.**

   If E be any event, then : $\mathbf{0 \leq P(E) \leq 1}$

EXERCISE 25(A)

1. A coin is tossed once. Find the probability of :
   (i) getting a tail.
   (ii) not getting a tail.
2. A bag contains 3 white, 5 black and 2 red balls, all of the same shape and size. A ball is drawn from the bag without looking into it, find the probability that the ball drawn is :
   (i) a black ball.
   (ii) a red ball.
   (iii) a white ball.
   (iv) not a red ball.
   (v) not a black ball.
3. In a single throw of a die, find the probability of getting a number :
   (i) greater than 4.
   (ii) less than or equal to 4.
   (iii) not greater than 4.
4. In a single throw of a die, find the probability that the number :
   (i) will be an even number.
   (ii) will not be an even number.
   (iii) will be an odd number.
5. From a well-shuffled deck of 52 playing-cards, one card is drawn. Find the probability that the card drawn will :
   (i) be a black card.
   (ii) not be a red card.
   (iii) be a red card.
   (iv) be a face card.
   (v) be a face card of red colour.
6. (i) If A and B are two complementary events then what is the relation between P(A) and P(B) ?
   (ii) If the probability of happening of an event A is 0·46. What will be the probability of not happening of the event A ?
7. In a T.T. match between Geeta and Ritu, the probability of the winning of Ritu is 0·73. Find the probability of :
   (i) winning of Geeta.
   (ii) not winning of Ritu.
8. In a race between Mahesh and John; the probability that John will lose the race is 0·54. Find the probability of :
   (i) winning of Mahesh.
   (ii) winning of John.
9. (i) Write the probability of a sure event.
   (ii) Write the probability of an event which is impossible.
   (iii) For an event E, write a relation representing the range of values of P(E).
10. In a single throw of a die, find the probability of getting :
    (i) 5
    (ii) 8
    (iii) a number less than 8.
    (iv) a prime number.
11. A die is thrown once. Find the probability of getting :
    (i) an even number.
    (ii) a number between 3 and 8.
    (iii) an even number or a multiple of 3.
12. Which of the following cannot be the probability of an event ?
    (i) $\frac{3}{5}$ (ii) 2·7 (iii) 43%
    (iv) −0·6 (v) −3·2 (vi) 0·35
13. A bag contains six identical black balls. A child withdraws one ball from the bag without looking into it. What is the probability that he takes out :
    (i) a white ball ? (ii) a black ball ?
14. A single letter is selected at random from the word 'Probability'. Find the probability that it is a vowel.
15. Ramesh chooses a date at random in January for a party (see the following figure).

JANUARY					
Mon.		6	13	20	27
Tue.		7	14	21	28
Wed.	1	8	15	22	29
Thu.	2	9	16	23	30
Fri.	3	10	17	24	31
Sat.	4	11	18	25	
Sun.	5	12	19	26	

Find the probability that he chooses :
(i) a Wednesday
(ii) a Friday
(iii) a Tuesday or a Saturday.

7 A die is thrown once. Find the probability of getting :
(i) an odd number (ii) a number greater than 4 (iii) a number between 2 and 6.

Solution :

In throwing a die once, the total possible outcomes are **six** *i.e.*, 1, 2, 3, 4, 5 and 6.

(i) **When an odd number is required *i.e.*** number required on the upper face is 1, 3 or 5. The total number of favourable cases is 3. [Any of 1, 3 and 5]

∴ **Probability of getting an odd number**

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{3}{6} = \frac{1}{2}$$ **Ans.**

(ii) **When a number greater than 4 is required *i.e.*** number required on the upper face is 5 or 6. The total number of favourable cases is 2. [Any of 5 and 6]

∴ **Probability of getting a number greater than 4**

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{2}{6} = \frac{1}{3}$$ **Ans.**

In short, **P(getting a number greater than 4)** $= \frac{2}{6} = \frac{1}{3}$ **Ans.**

(iii) **When a number between 2 and 6 is required *i.e.*** number required on the upper face is 3, 4 or 5.

The total number of favourable cases = 3

∴ **P(getting a number between 2 and 6)**

$$= \frac{\text{No. of favourable outcomes}}{\text{All possible outcomes}} = \frac{3}{6} = \frac{1}{2}$$ **Ans.**

25.4 Important :

1. **In tossing a coin once :**
   The total possible outcomes are : head (H) and tail (T).
   And, the number of these possible outcomes = $2^1 = 2$
2. **In tossing a coin two times** or in tossing of two coins simultaneously :
   The total number of possible outcomes = $2^2 = 4$
   And, the possible outcomes are :
   (i) Head on the first coin and head on the second *i.e.* HH,
   (ii) Head on the first coin and tail on the second *i.e.* HT,
   (iii) Tail on the first coin and head on the second *i.e.* TH and
   (iv) Tail on the first coin and tail on the second *i.e.* TT.

In other words; the total possible outcomes are : HH, HT, TH and TT.

Tossing of two coins simultaneously or tossing one coin twice, gives the same outcomes. Similarly, tossing of three coins simultaneously or tossing one coin three times gives the same outcomes.

3. **In tossing a coin three times** or in tossing three coins simultaneously :
The total number of possible outcomes = $2^3 = 8$
And the possible outcomes are : HHH, HHT, HTH, THH, HTT, THT, TTH and TTT.

In general, if a coin is tossed n times or n coins are tossed simultaneously, the number of all possible outcomes = 2^n.

In the same way :

4. **In throwing** (rolling) **a die once** :
The total possible outcomes are : 1, 2, 3, 4, 5 and 6.
And the number of these outcomes = $6^1 = 6$.

5. **In rolling** (throwing) **two dice simultaneously** or one die two times :
The total number of outcomes is = $6^2 = 6 \times 6 = 36$
And the possible outcomes are :
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5) and (6, 6).

1. (3, 5) means : 3 on the first die and 5 on the second dice. (6, 4) means : 6 on the first dice and 4 on the second dice and so on.
2. Rolling a dice two times gives the same result as rolling two dice simultaneously. Similarly, rolling a dice three times gives the same result as rolling three dice simultaneously.
3. In general, if a dice is rolled n times or n-dice are rolled simultaneously, the total number of outcomes = 6^n.

8 Two dice are thrown simultaneously. Find the probability that :
(i) both the dice show the same number.
(ii) the first dice shows 6.
(iii) the total (sum) of the numbers on the dice is 9.
(iv) the product of the numbers on the dice is 8.
(v) the total of the numbers on the dice is greater than 9.

Solution :

When two dice are thrown simultaneously, all the possible outcomes are as shown alongside :

Clearly, the total number of all possible outcomes = $6^2 = 6 \times 6 = 36$.

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6).

(i) **When both the dice show the same number :**

The favourable outcomes are : (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6)

$\Rightarrow$ The number of favourable outcomes = 6

$\therefore$ **Required probability** = $\frac{6}{36} = \frac{1}{6}$ **Ans.**

(ii) **When the first die shows 6 :**

The favourable outcomes are : (6, 1), (6, 2), (6, 3), (6, 4), (6, 5) and (6, 6).

$\Rightarrow$ The number of favourable outcomes = 6

$\therefore$ **Required probability** = $\frac{6}{36} = \frac{1}{6}$ **Ans.**

(iii) **When the total of the numbers on the dice is 9 :**

The favourable outcomes are : (3, 6), (4, 5), (5, 4) and (6, 3)

$\Rightarrow$ The number of favourable outcomes = 4

$\therefore$ **Required probability** = $\frac{4}{36} = \frac{1}{9}$ **Ans.**

(iv) **When the product of the numbers on the dice is 8 :**

The favourable outcomes are : (2, 4) and (4, 2)

$\Rightarrow$ The number of favourable outcomes = 2

$\therefore$ **Required probability** = $\frac{2}{36} = \frac{1}{18}$ **Ans.**

(v) **When the total of the numbers on the dice is greater than 9 :**

The favourable outcomes are : (4, 6), (5, 5), (5, 6), (6, 4), (6, 5) and (6, 6).

$\Rightarrow$ The number of favourable outcomes = 6

$\therefore$ **Required probability** = $\frac{6}{36} = \frac{1}{6}$ **Ans.**

9 A card is drawn from a pack of 100 cards numbered 1 to 100. Find the probability of drawing a number which is a perfect square.

Solution :

Total number of all possible outcomes = 100

Since required (favourable) outcomes are : 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100

$\therefore$ The number of favourable outcomes = 10

$\Rightarrow$ **Required probability** = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$= \frac{10}{100} = \frac{1}{10}$ **Ans.**

10 Three identical coins are tossed together. What is the probability of obtaining :

(i) all heads ? (ii) exactly two heads ? (iii) exactly one head ?

(iv) at least one head ? (v) at least two heads ? (vi) all tails ?

Solution :

When three coins are tossed together (or, a single coin is tossed three times); the possible outcomes are : HHH, HHT, HTH, THH, HTT, THT, TTH and TTT

i.e. the total number of possible outcomes = 8

For the favourable outcomes, we can form a table as shown below :

Favourable outcomes	No. of favourable outcomes
(i) All heads : HHH	1
(ii) Exactly two heads : HHT, HTH, THH	3
(iii) Exactly one head : HTT, THT, TTH	3
(iv) At least one head : HTT, THT, TTH, HHT, HTH, THH, HHH	7
(v) At least two heads : HHT, HTH, THH, HHH	4
(vi) All tails : TTT	1

(i) **P (all heads) = $\frac{1}{8}$** **Ans.**

(ii) **P (exactly two heads) = $\frac{3}{8}$** **Ans.**

(iii) **P (exactly one head) = $\frac{3}{8}$** **Ans.**

(iv) **P (at least one head) = $\frac{7}{8}$** **Ans.**

(v) **P (at least two heads) = $\frac{4}{8} = \frac{1}{2}$** **Ans.**

(vi) **P (all tails) = $\frac{1}{8}$** **Ans.**

11 Two dice are rolled simultaneously. Find the probability of :

(i) obtaining a total of at least 9.

(ii) getting a multiple of 2 on one die and a multiple of 3 on the other die.

(iii) getting a multiple of 3 as the sum.

Solution :

When two dice are rolled simultaneously; the total number of possible outcomes = $6 \times 6 = 36$

(i) **For obtaining a total of at least 9;** the favourable outcomes are :
(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5) and (6, 6)

$\Rightarrow$ Number of favourable outcomes = 10

$\Rightarrow$ **Required probability** = $\frac{10}{36} = \frac{5}{18}$ **Ans.**

(ii) **For getting a multiple of 2 on one die and a multiple of 3 on the other die;** the favourable outcomes are : (2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6), (6, 4), (6, 2), (3, 6), (3, 4) and (3, 2)

$\Rightarrow$ Number of favourable outcomes = 11

$\Rightarrow$ **Required probability** = $\frac{11}{36}$ **Ans.**

(iii) **For getting multiple of 3 as the sum** *i.e.* to get the sum of numbers on both the dice as 3, 6, 9 or 12.

Clearly, favourable outcomes are : (1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3) and (6, 6)

$\Rightarrow$ Number of favourable outcomes = 12

$\Rightarrow$ **Required probability** = $\frac{12}{36} = \frac{1}{3}$ **Ans.**

EXERCISE 25(B)

1. Nine cards (identical in all respects) are numbered 2 to 10. A card is selected from them at random. Find the probability that the card selected will be :
   (i) an even number.
   (ii) a multiple of 3.
   (iii) an even number and a multiple of 3.
   (iv) an even number or a multiple of 3.
2. Hundred identical cards are numbered from 1 to 100. The cards are well shuffled and then a card is drawn. Find the probability that the number on the card drawn is :
   (i) a multiple of 5.
   (ii) a multiple of 6.
   (iii) between 40 and 60.
   (iv) greater than 85.
   (v) less than 48.
3. From 25 identical cards, numbered 1, 2, 3, 4, 5 . . . . ., 24, 25; one card is drawn at random. Find the probability that the number on the card drawn is a multiple of :
   (i) 3 (ii) 5 (iii) 3 and 5 (iv) 3 or 5.
4. A die is thrown once. Find the probability of getting a number :
   (i) less than 3.
   (ii) greater than or equal to 4.
   (iii) less than 8.
   (iv) greater than 6.
5. A book contains 85 pages. A page is chosen at random. What is the probability that the sum of the digits on the page is 8 ?
6. A pair of dice is thrown. Find the probability of getting a sum of 10 or more, if 5 appears on the first die.

   Total number of cases = 6 × 6 = 36.
   Since favourable cases are (5, 5) and (5, 6).
   ∴ No. of favourable cases = 2

7. If two coins are tossed once, what is the probability of getting :
   (i) 2 heads ?
   (ii) at least one head ? **[2012]**
   (iii) both heads or both tails ?
8. Two dice are rolled together. Find the probability of getting :
   (i) a total of at least 10.
   (ii) a multiple of 2 on one die and an odd number on the other die.
9. A card is drawn from a well-shuffled pack of 52 cards. Find the probability that the card drawn is :
   (i) a spade. (ii) a red card.
   (iii) a face card. (iv) 5 of heart or diamond.
   (v) Jack or queen. (vi) ace and king.
   (vii) a red and a king.
   (viii) a red or a king.
10. A bag contains 16 coloured balls. Six are green, 7 are red and 3 are white. A ball is chosen, without looking into the bag. Find the probability that the ball chosen is :
    (i) red (ii) not red
    (iii) white (iv) not white
    (v) green or red (vi) white or green
    (vii) green or red or white.
11. A ball is drawn at random from a box containing 12 white, 16 red and 20 green balls. Determine the probability that the ball drawn is :
    (i) white (ii) red
    (iii) not green (iv) red or white.
12. A card is drawn from a pack of 52 cards. Find the probability that the card drawn is :
    (i) a red card (ii) a black card
    (iii) a spade (iv) an ace
    (v) a black ace (vi) ace of diamonds
    (vii) not a club (viii) a queen or a jack.
13. Thirty identical cards are marked with numbers 1 to 30. If one card is drawn at random, find the probability that it is :
    (i) a multiple of 4 or 6.
    (ii) a multiple of 3 and 5.
    (iii) a multiple of 3 or 5.

14. In a single throw of two dice, find the probability of :
    (i) a doublet
    (ii) a number less than 3 on each dice.
    (iii) an odd number as a sum.
    (iv) a total of atmost 10.
    (v) an odd number on one dice and a number less than or equal to 4 on the other dice.

EXERCISE 25(C)

1. A bag contain 3 red balls, 4 blue balls and one yellow ball, all the balls being identical in shape and size. If a ball is taken out of the bag without looking into it; find the probability that the ball is :
    (i) yellow (ii) red
    (iii) not yellow (iv) neither yellow nor red
2. A dice is thrown once. What is the probability of getting a number :
    (i) greater than 2 ?
    (ii) less than or equal to 2 ?
3. From a well-shuffled deck of 52 cards, one card is drawn. Find the probability that the card drawn is :
    (i) a face card.
    (ii) not a face card.
    (iii) a queen of black colour.
    (iv) a card with number 5 or 6.
    (v) a card with number less than 8.
    (vi) a card with number between 2 and 9.
4. In a match between A and B;
    (i) the probability of winning of A is 0·83. What is the probability of winning of B?
    (ii) the probability of losing the match is 0·49 for B. What is the probability of winning of A ?
5. A and B are friends. Ignoring the leap year, find the probability that both friends will have:
    (i) different birthdays.
    (ii) the same birthday.

    (i) Out of A and B, one may have his birthday on any of the 365 days of the year. But the other can not have his birthday on the same day.
    Hence, the no. of favourable outcomes
    $= 365 - 1 = 364.$
    $\Rightarrow$ **Req. prob.** $= \frac{364}{365}$ **Ans.**

    (ii) **Req. prob.** $= 1 - \frac{364}{365} = \frac{1}{365}$ **Ans.**

6. A man tosses two different coins (one of ₹ 2 and another of ₹ 5) simultaneously. What is the probability that he gets :
    (i) at least one head ? (ii) atmost one head ?
7. A box contains 7 red balls, 8 green balls and 5 white balls. A ball is drawn at random from the box. Find the probability that the ball is :
    (i) white (ii) neither red nor white.
8. All the three face cards of spades are removed from a well-shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting :
    (i) a black face card
    (ii) a queen
    (iii) a black card.
9. In a musical chairs game, a person has been advised to stop playing the music at any time within 40 seconds after its start.
    What is the probability that the music will stop within the first 15 seconds ?

    The favourable results = 0 sec to 15 sec and, the total results = 0 sec to 40 sec.
    So, we can say,
    The favourable outcomes = 15 and, the total no. of outcomes = 40
    $\Rightarrow$ **Req. prob.** $= \frac{15}{40} = \frac{3}{8}$ **Ans.**

10. In a bundle of 50 shirts, 44 are good, 4 have minor defects and 2 have major defects. What is the probability that :
    (i) it is acceptable to a trader who accepts only a good shirt ?
    (ii) it is acceptable to a trader who rejects only a shirt with major defects ?
11. Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is :
    (i) 8 (ii) 13
    (iii) less than or equal to 12.
12. Which of the following cannot be the probability of an event ?
    (i) $\frac{3}{7}$ (ii) 0·82
    (iii) 37% (iv) – 2·4
13. If P(E) = 0·59; find P(not E).

14. A bag contains a certain number of red balls. A ball is drawn. Find the probability that the ball drawn is :
    (i) black (ii) red.

15. The probability that two boys do not have the same birthday is 0·897. What is the probability that the two boys have the same birthday ?

16. A bag contains 10 red balls, 16 white balls and 8 green balls. A ball is drawn out of the bag at random. What is the probability that the ball drawn will be :
    (i) not red ?
    (ii) neither red nor green ?
    (iii) white or green ?

17. A bag contains twenty ₹ 5 coins, fifty ₹ 2 coins and thirty ₹ 1 coins. If it is equally likely that one of the coins will fall down when the bag is turned upside down, what is the probability that the coin :
    (i) will be a ₹ 1 coin ?
    (ii) will not be a ₹ 2 coin ?
    (iii) will neither be a ₹ 5 coin nor be a ₹ 1 coin ?

18. A game consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; as shown below.

    If the outcomes are equally likely, find the probability that the pointer will point at :
    (i) 6 (ii) an even number.
    (iii) a prime number.
    (iv) a number greater than 8.
    (v) a number less than or equal to 9.
    (vi) a number between 3 and 11.

19. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting :
    (i) a queen of red colour.
    (ii) a black face card.
    (iii) the jack or the queen of hearts.
    (iv) a diamond.
    (v) a diamond or a spade.

20. From a deck of 52 cards, all the face cards are removed and then the remaining cards are shuffled. Now one card is drawn from the remaining deck. Find the probability that the card drawn is :
    (i) a black card. (ii) 8 of red colour.
    (iii) a king of black colour.

21. Seven cards :- the eight, the nine, the ten, jack, queen, king and ace of diamonds are well shuffled. One card is then picked up at random.
    (i) What is the probability that the card drawn is the eight or the king ?
    (ii) If the king is drawn and put aside, what is the probability that the second card picked up is :
        (a) an ace ? (b) a king ?

22. A box contains 150 bulbs out of which 15 are defective. It is not possible to just look at a bulb and tell whether or not it is defective. One bulb is taken out at random from this box. Calculate the probability that the bulb taken out is :
    (i) a good one
    (ii) a defective one.

23. (i) 4 defective pens are accidentally mixed with 16 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is drawn at random from the lot. What is the probability that the pen is defective ?
    (ii) Suppose the pen drawn in (i) is defective and is not replaced. Now one more pen is drawn at random from the rest. What is the probability that this pen is :
        (a) defective ?
        (b) not defective ?

24. A bag contains 100 identical marble stones which are numbered from 1 to 100. If one stone is drawn at random from the bag, find the probability that it bears :
    (i) a perfect square number.
    (ii) a number divisible by 4.
    (iii) a number divisible by 5.
    (iv) a number divisible by 4 or 5.
    (v) a number divisible by 4 and 5.

25. A circle with diameter 20 cm is drawn somewhere on a rectangular piece of paper with length 40 cm and width 30 cm. This paper is kept horizontal on table top and a die, very small in size, is dropped on the rectangular paper without seeing towards it. If the die falls and lands on the paper only, find the probability that it will fall and land :
    (i) inside the circle.
    (ii) outside the circle.

26. Two dice (each bearing numbers 1 to 6) are rolled together. Find the probability that the sum of the numbers on the upper-most faces of two dice is :

(i) 4 or 5. (ii) 7, 8 or 9.

(iii) between 5 and 8. (iv) more than 10.

(v) less than 6.

27. Three coins are tossed together. Write all the possible outcomes. Now, find the probability of getting :

(i) exactly two heads. (ii) at least two heads.

(iii) atmost two heads. (iv) all tails.

(v) at least one tail.

28. Two dice are thrown simultaneously. What is the probability that :

(i) 4 will not come up either time ?

(ii) 4 will come up at least once ?

Throwing two dice simultaneously or one dice twice give the same results.

29. Cards marked with numbers 1, 2, 3, 4, ... 20 are well shuffled and a card is drawn at random. What is the probability that the number on the card is :

(i) a prime number ? (ii) divisible by 3 ?

(iii) a perfect square ? **[2010]**

30. Offices in Delhi are open for five days in a week (Monday to Friday). Two employees of an office remain absent for one day in the same particular week. Find the probability that they remain absent on :

(i) the same day (ii) consecutive day

(iii) different days.

Total number of possible outcomes = $5 \times 5 = 25$. Let the five days of the week be denoted as Monday by M, Tuesday by T, Wednesday by W, Thursday by Th and Friday by F; then :

(i) Favourable cases are : MM, TT, WW, Th Th and F F *i.e.* 5 in all.

$\therefore$ **Req. probability** $= \frac{5}{25} = \frac{1}{5}$ **Ans.**

(ii) Favourable outcomes are : M T, T M, T W, W T, W Th, Th W, Th F and F Th *i.e.* 8 in all.

$\therefore$ **Req. probability** $= \frac{8}{25}$ **Ans.**

(iii) **P(absent on different days)**

$= 1 - \text{P(absent on the same days)}$

$= 1 - \frac{1}{5} = \frac{4}{5}$.

31. A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball; find the number of black balls in the box. **[2013]**

Let the box contains x black balls.

$\therefore$ Total number of balls in the box $= x + 30$

Probability of drawing a

black ball $= \frac{x}{x + 30}$

and probability of drawing a

white ball is $\frac{30}{x + 30}$

Given : $\frac{x}{x + 30} = \frac{2}{5} \times \frac{30}{x + 30}$

$\Rightarrow$ $x = 12$ **Ans.**

32. From a pack of 52 playing cards, all cards whose numbers are multiples of 3 are removed. A card is now drawn at random.

What is the probability that the card drawn is

(i) a face card (King, Jack or Queen)

(ii) an even numbered red card ? **[2011]**

33. A die has 6 faces marked by the given numbers as shown below :

1	2	3	–1	–2	–3

The die is thrown once. What is the probability of getting

(i) a positive integer ?

(ii) an integer greater than –3 ?

(iii) the smallest integer ? **[2014]**

34. A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is :

(i) a green ball.

(ii) a white or a red ball.

(iii) neither a green ball nor a white ball.

[2015]